



基于图形计算器

开展对正整数立方和的探究性学习

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1. 在数的阵列队形中摸索规律

$$1^3$$

$$2^3$$

$$3^3$$

$$4^3$$



1. 在数的阵列队形中摸索规律

$$1^3 \longrightarrow 1 \times 1$$

$$2^3 \longrightarrow 2 \times 4$$

$$3^3 \longrightarrow 3 \times 9$$

$$4^3 \longrightarrow 4 \times 16$$



1. 在数的阵列队形中摸索规律

$$1^3 \longrightarrow 1 \times 1$$

1

$$2^3 \longrightarrow 2 \times 4$$

3

5

$$3^3 \longrightarrow 3 \times 9$$

7

9

11

$$4^3 \longrightarrow 4 \times 16$$

13

15

17

19



(1) 三角形数阵

1.12 1.13 1.14 *Sum of cubes RAD

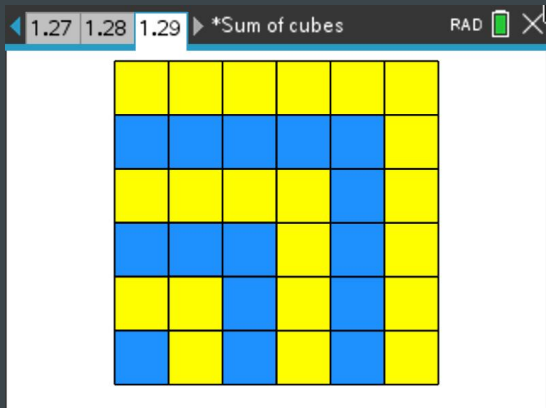
	A	B	C	D	E	F	G
=	=seq(n						
1	1	1					
2	8	3	5				
3	27	7	9	11			
4	64	13	15	17	19		
5	125	21	23	25	27	29	
A	cubes:=seq(n ³ ,n,1,10)						

1.12 1.13 1.14 *Sum of cubes RAD

$$\sum_{i=1}^n \left(\sum_{j=1}^i \left(2 \cdot \frac{i \cdot (i-1)}{2} - 1 + 2 \cdot j \right) \right)$$

$$\frac{n^2 \cdot (n+1)^2}{4}$$

(1) 三角形数阵



1.14 1.15 1.16 ▶ *Sum of cubes RAD

$$\left(\sum_{k=1}^n (k) \right)^2 = \frac{n^2 \cdot (n+1)^2}{4}$$

|

从 1 开始的连续 $\frac{n(n+1)}{2}$ 个奇数的和

(2) 正方形数阵

1.15 1.16 1.17 *Sum of cubes RAD

	A	B	C	D	E	F
=						
1	1	2	3	4	5	
2	2	4	6	8	10	
3	3	6	9	12	15	
4	4	8	12	16	20	
5	5	10	15	20	25	
F6						

1.16 1.17 1.18 *Sum of cubes RAD

$$\sum_{i=1}^n \left(\sum_{j=1}^n (i \cdot j) \right) = \frac{n^2 \cdot (n+1)^2}{4}$$

(3) 矩形数组

1.17 1.18 1.19 *Sum of cubes RAD

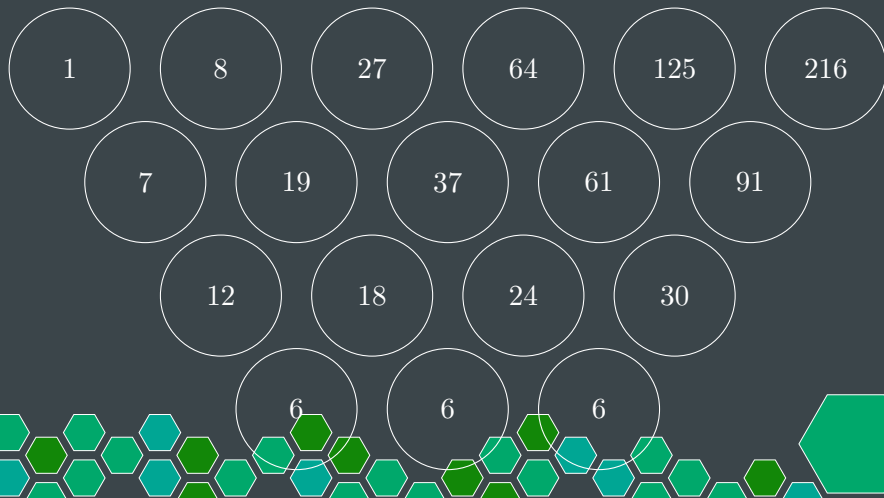
	A	B	C	D	E	F	G
=							
1	1	1	1	1	1	1	
2	4	4	4	4	4	4	
3	9	9	9	9	9	9	
4	16	16	16	16	16	16	
5	25	25	25	25	25	25	

1.18 1.19 1.20 *Sum of cubes RAD

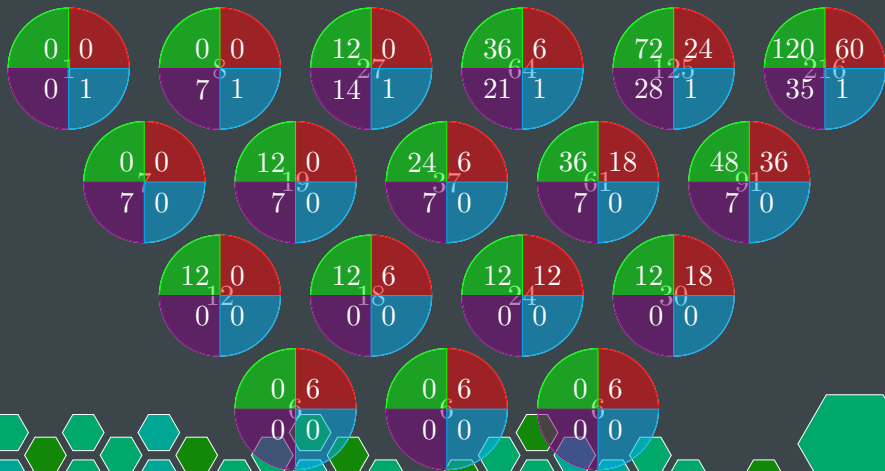
$$(n+1) \cdot \sum_{k=1}^n (k^2) - \sum_{i=1}^n \left(\sum_{j=1}^i (j^2) \right)$$

$$\frac{n^2 \cdot (n+1)^2}{4}$$

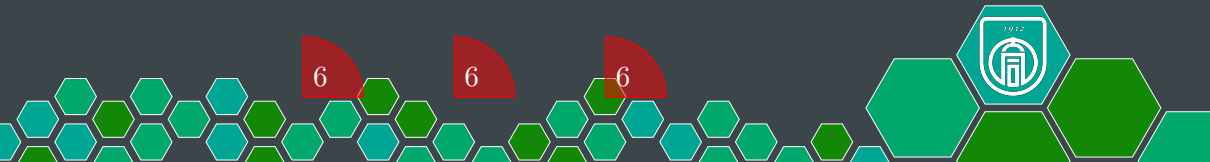
(4) 差分表



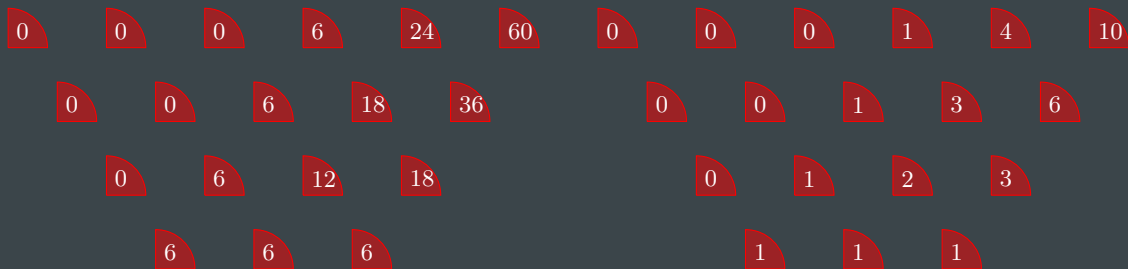
(4) 差分表



(4) 差分表



(4) 差分表



$\times 1$ $\xrightarrow{\text{分拆}}$ $\times 6$



(4) 差分表



(4) 差分表



$\times 1$ 分拆 $\rightarrow \times 12$



(4) 差分表

0

7

14

21

28

35

7

7

7

7

7

0

0

0

0

0

0

0



(4) 差分表



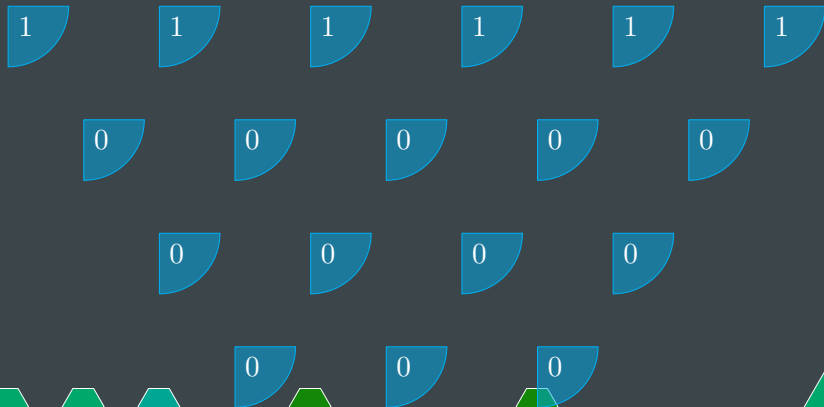
$\times 1$

分拆

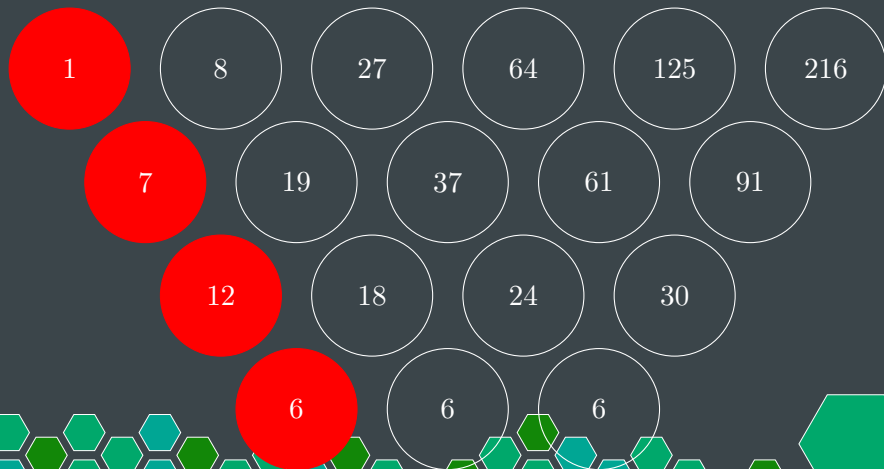
$\times 7$



(4) 差分表



(4) 差分表



(4) 差分表

1.19 1.20 1.21 *Sum of cubes RAD

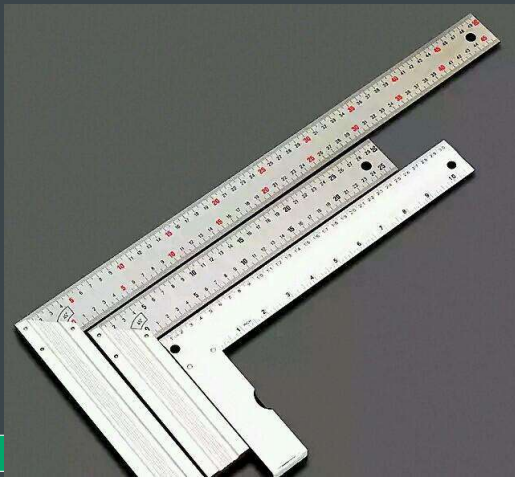
	A	B	C	D	E	F
=	=seq(r					
1	1	7	12	6	0	
2	8	19	18	6	0	
3	27	37	24	6	0	
4	64	61	30	6	0	
5	125	91	36	6	0	

C5 =b6-b5

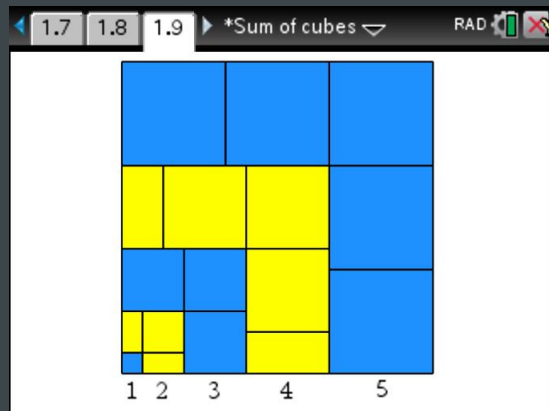
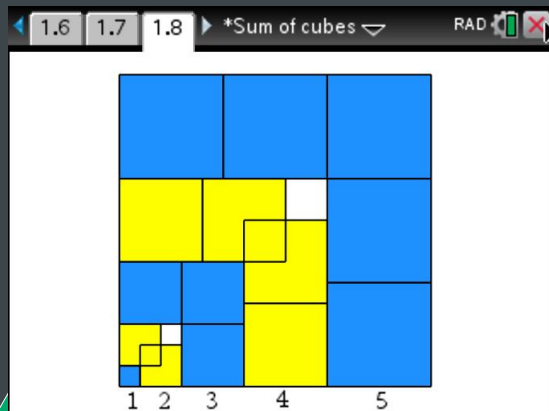
1.20 1.21 1.22 *Sum of cubes RAD

$$\frac{6 \cdot nCr(n,4) + 12 \cdot nCr(n,3) + 7 \cdot nCr(n,2) + 1 \cdot nCr(n,1)}{4}$$

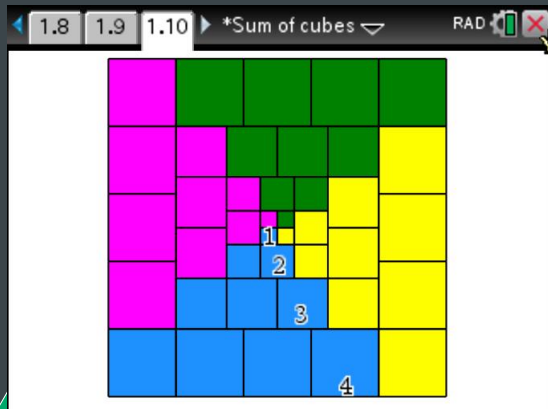
2. 在图形拼接中探究摸索



(5) 角尺拼图一、二



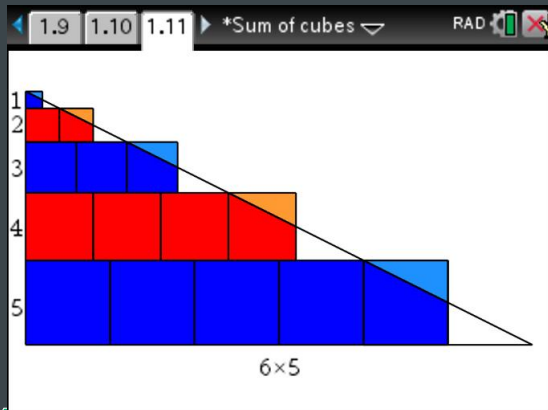
(7) 旋转拼图



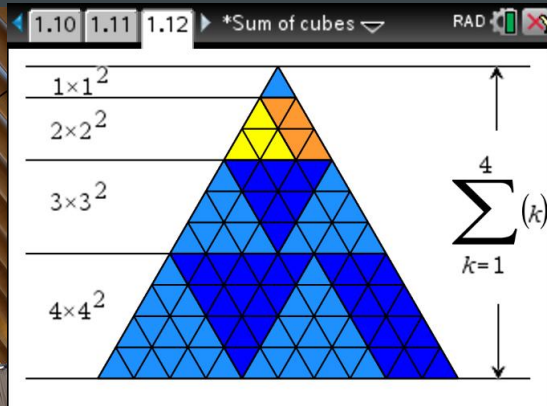
$$\sum_{k=1}^n k^3 = \frac{(n(n+1))^2}{4}$$



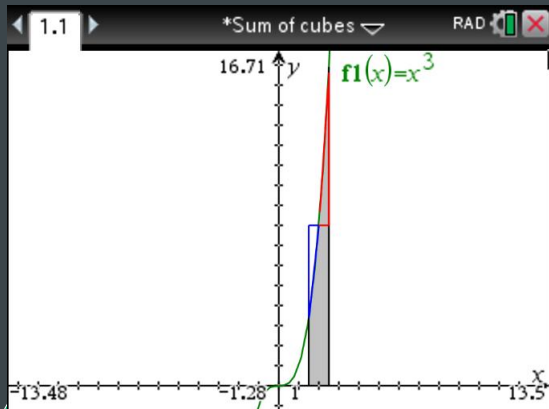
(8) (割补后) 三角形拼图



(9) 等边三角形拼图



3. 借助技术实现别样想法



多出一块

$$\int_k^{k+\frac{1}{2}} x^3 dx - \frac{k^3}{2}$$

少掉一块

$$\frac{k^3}{2} - \int_{k-\frac{1}{2}}^k x^3 dx$$



(10) 积分思想

The image shows three sequential TI-Nspire CAS calculator screens illustrating the integral approach to finding the area of a curved trapezoid.

- Screen 1 (Left):** Shows the integral of x^3 from $k - \frac{1}{2}$ to $k + \frac{1}{2}$. The result is $\frac{-k}{4}$.
- Screen 2 (Middle):** Shows the integral of x^3 from $\frac{1}{2}$ to $n + \frac{1}{2}$. The result is $\frac{n^2 \cdot (n^2 + 2 \cdot n + 1)}{4}$.
- Screen 3 (Right):** Shows the summation of the integral from Screen 1 over k from 1 to n . The result is $\frac{n^2 \cdot (n+1)^2}{4}$.

曲边梯形相较于矩形面积多了 $\frac{k}{4}$

TI-Nspire 的计算机代数系统 (CAS)



(11) 导数思想

<p>Define $f(x)=a \cdot x^4+b \cdot x^3+c \cdot x^2+d \cdot x+e$ Done</p> <p>Define $g(x)=f(x)-f(x-1)-x^3$ Done</p> <p>solve $\left(f(1)=1 \text{ and } g(1)=0 \text{ and } \frac{d}{dx}(g(x))=0 \text{ and } \right.$ $\left. x=1 \text{ and } a=\frac{1}{4} \text{ and } b=\frac{1}{2} \text{ and } c=\frac{1}{4} \text{ and } d=0 \text{ and } \right.$</p>	<p>solve $\left(f(1)=1 \text{ and } g(1)=0 \text{ and } \frac{d}{dx}(g(x))=0 \mid \right.$ $\text{and } \frac{d^2}{dx^2}(g(x))=0 \text{ and } \frac{d^3}{dx^3}(g(x))=0$ $\left. \text{and } \frac{d^4}{dx^4}(g(x))=0 \text{ and } x=1, \{x,a,b,c,d,e\} \right)$</p>	<p>$x=1 \text{ and } a=\frac{1}{4} \text{ and } b=\frac{1}{2} \text{ and } c=\frac{1}{4} \text{ and } d=0 \mid$ $\text{and } e=0$</p>
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适当定义新函数

导数功能与求解方程组功能



4. 大胆尝试技术验证

- 运用累加化归
- 运用 Abel 变换化归
- 运用二项式定理化归
- 运用组合数性质二化归
- 裂项相消



化归

$$\sum_{k=1}^n k(k+1)$$

$$\sum_{k=1}^n k$$

$$\sum_{k=1}^n k^2$$

$$\sum_{k=1}^n (k-1)k(k+1)$$

$$\sum_{k=1}^n k(k+1)(k+2)$$

$$\sum_{k=1}^n k^3$$

$$\sum_{k=1}^n k^2 \frac{(k+1)^2 - (k-1)^2}{4}$$



参考文献

- [1] 徐希来. 中学数学课堂教学中提高记忆效能策略的研究 [D]. 上海: 华东师范大学,2018:67-71.
- [2] 张礼恩. 对正整数立方和公式推导的赏析 [J]. 上海中学数学.2012,(11):43-45.



Thank you!

I am reachable at xuxilai2003@163.com.

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