基于图形计算器

开展对正整数立方和的探究性学习

June 12th 2019

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目录

1．在数的阵列队形中摸索规律

1^3

2^3

3^3

4^3

1. 在数的阵列队形中摸索规律

$1^3 \longrightarrow 1 \times 1$

$2^3 \longrightarrow 2 \times 4$

$3^3 \longrightarrow 3 \times 9$

$4^3 \longrightarrow 4 \times 16$

1. 在数的阵列队形中摸索规律

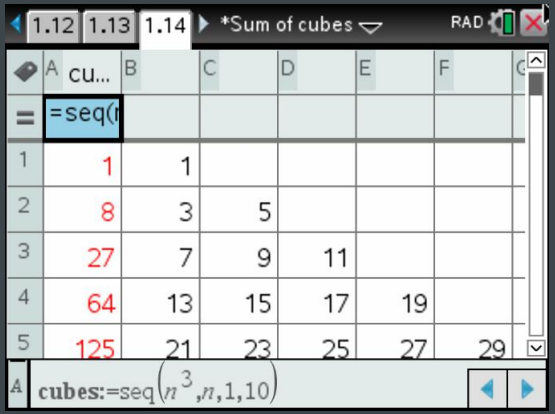
$1^3 \longrightarrow 1 \times 1$ 1

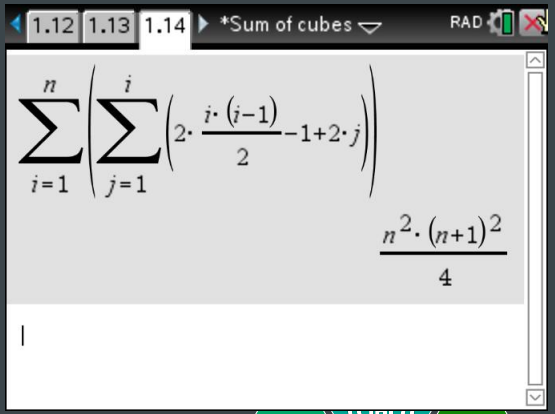
$2^3 \longrightarrow 2 \times 4$ 3 5

$3^3 \longrightarrow 3 \times 9$ 7 9 11

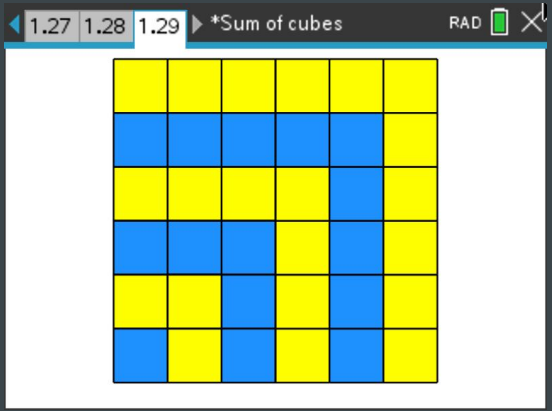
$4^3 \longrightarrow 4 \times 16$ 13 15 17 19

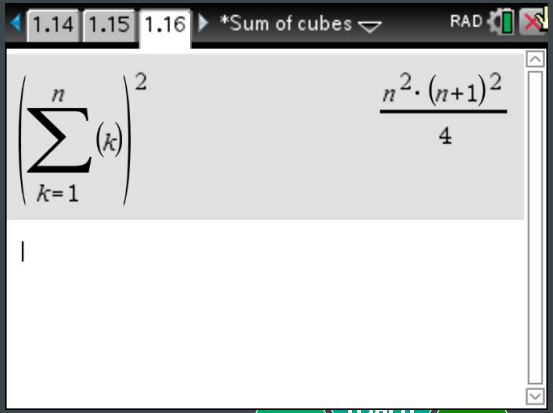
（1）三角形数阵


1.12 1.13 1.14 *Sum of cubes RAD
A cu... B C D E F G
=seq(
1 1
2 8 3 5
3 27 7 9 11
4 64 13 15 17 19
5 125 21 23 25 27 29
cubes:=seq(n^3,n,1,10)


1.12 1.13 1.14 *Sum of cubes RAD
$\sum_{i=1}^{n}\left(\sum_{j=1}^{i}\left(2\cdot\frac{i\cdot(i-1)}{2}-1+2\cdot j\right)\right)$
$\frac{n^2\cdot(n+1)^2}{4}$

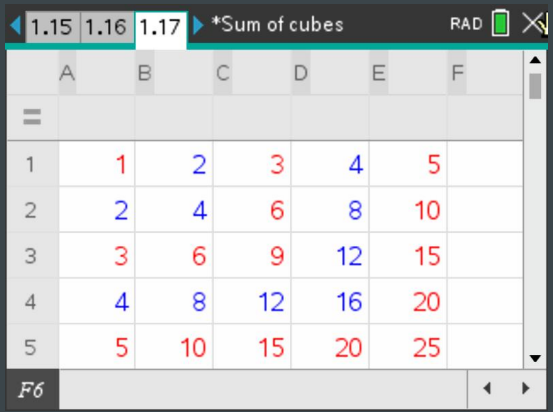
(1) 三角形数阵

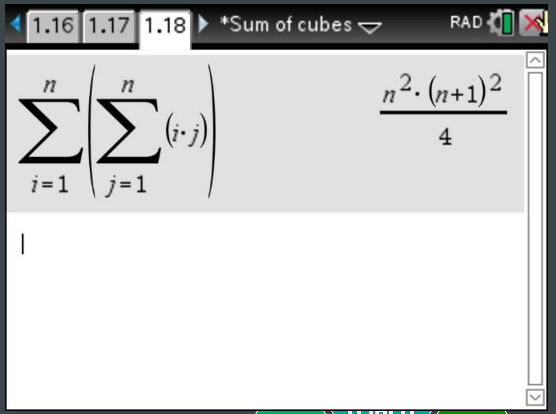




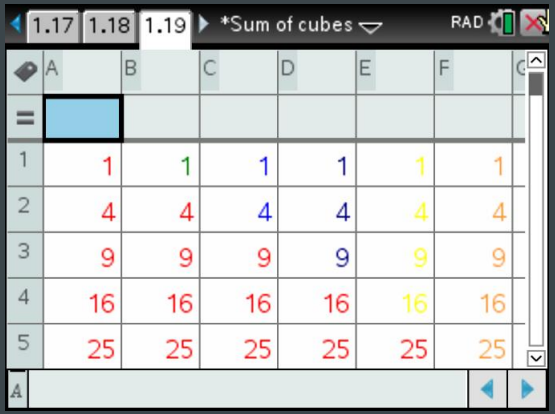
从 1 开始的连续 $\frac{n(n+1)}{2}$ 个奇数的和

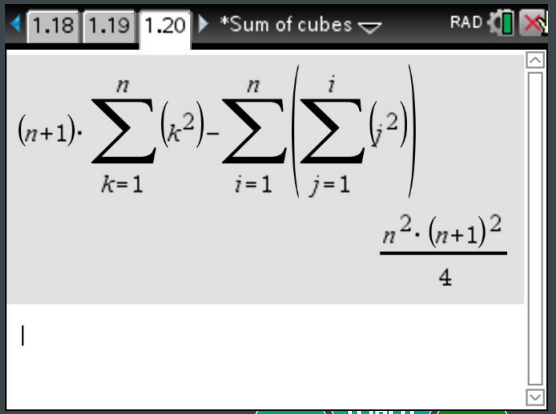
(2) 正方形数阵


1.15 1.16 1.17 *Sum of cubes RAD
A B C D E F
1 1 2 3 4 5
2 2 4 6 8 10
3 3 6 9 12 15
4 4 8 12 16 20
5 5 10 15 20 25
F6

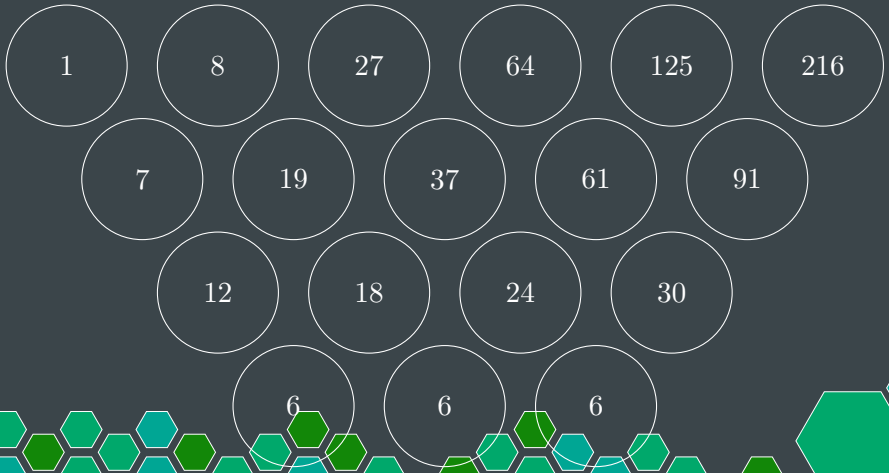

1.16 1.17 1.18 *Sum of cubes RAD
$\sum_{i=1}^{n}\left(\sum_{j=1}^{n}(i\cdot j)\right)$
$\frac{n^2\cdot(n+1)^2}{4}$

(3) 矩形数阵

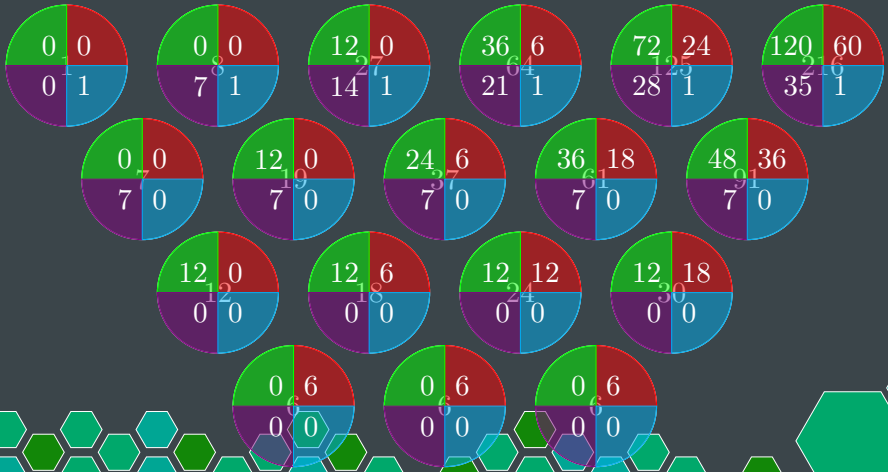

1.17 1.18 1.19 *Sum of cubes RAD
A B C D E F
1 1 1 1 1 1 1
2 4 4 4 4 4 4
3 9 9 9 9 9 9
4 16 16 16 16 16 16
5 25 25 25 25 25 25


1.18 1.19 1.20 *Sum of cubes RAD
$(n+1)\cdot\sum_{k=1}^{n}(k^2)-\sum_{i=1}^{n}\left(\sum_{j=1}^{i}(j^2)\right)$
$\frac{n^2\cdot(n+1)^2}{4}$

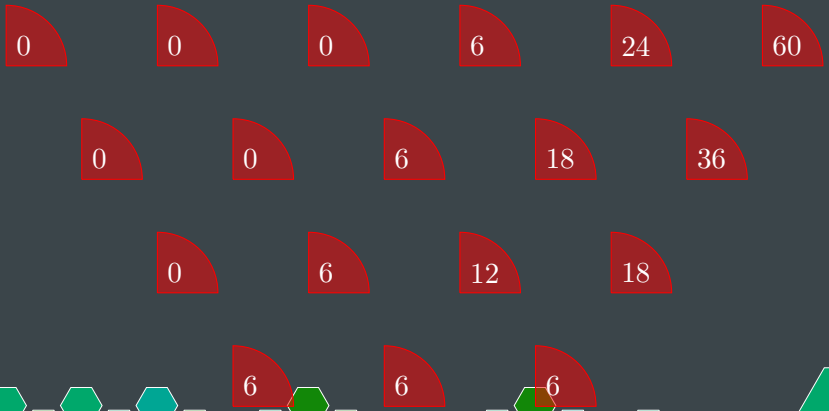
(4) 差分表



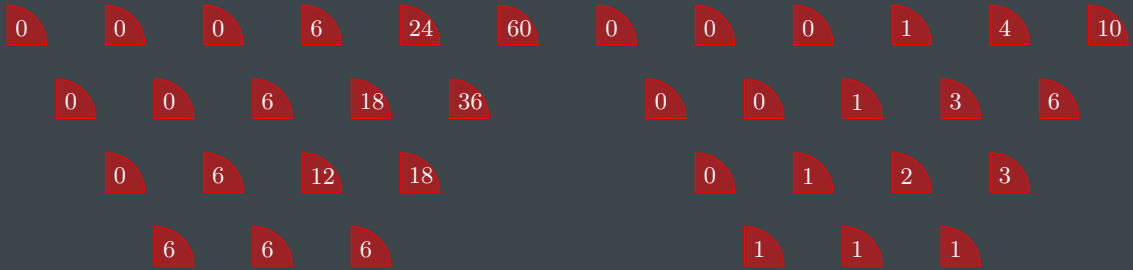
(4) 差分表


0 0
1
0 1
0 0
8
7 1
12 0
27
14 1
36 6
64
21 1
72 24
125
28 1
120 60
216
35 1
0 0
7
7 0
12 0
19
7 0
24 6
37
7 0
36 18
61
7 0
48 36
91
7 0
12 0
12
0 0
12 6
18
0 0
12 12
24
0 0
12 18
30
0 0
0 6
6
0 0
0 6
6
0 0
0 6
6
0 0

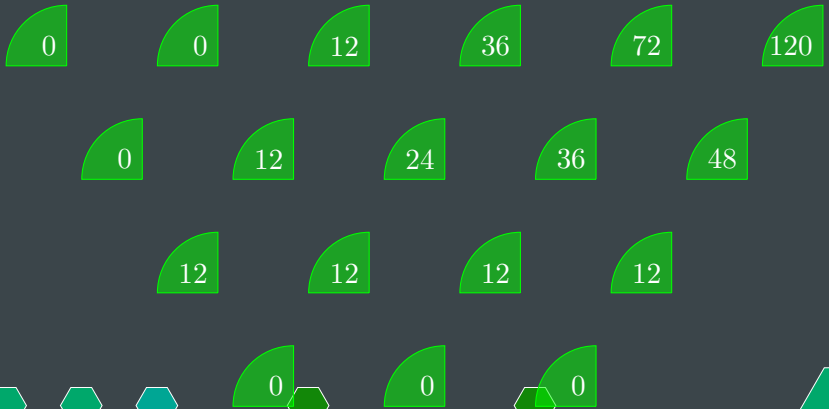
(4) 差分表



(4) 差分表



(4) 差分表

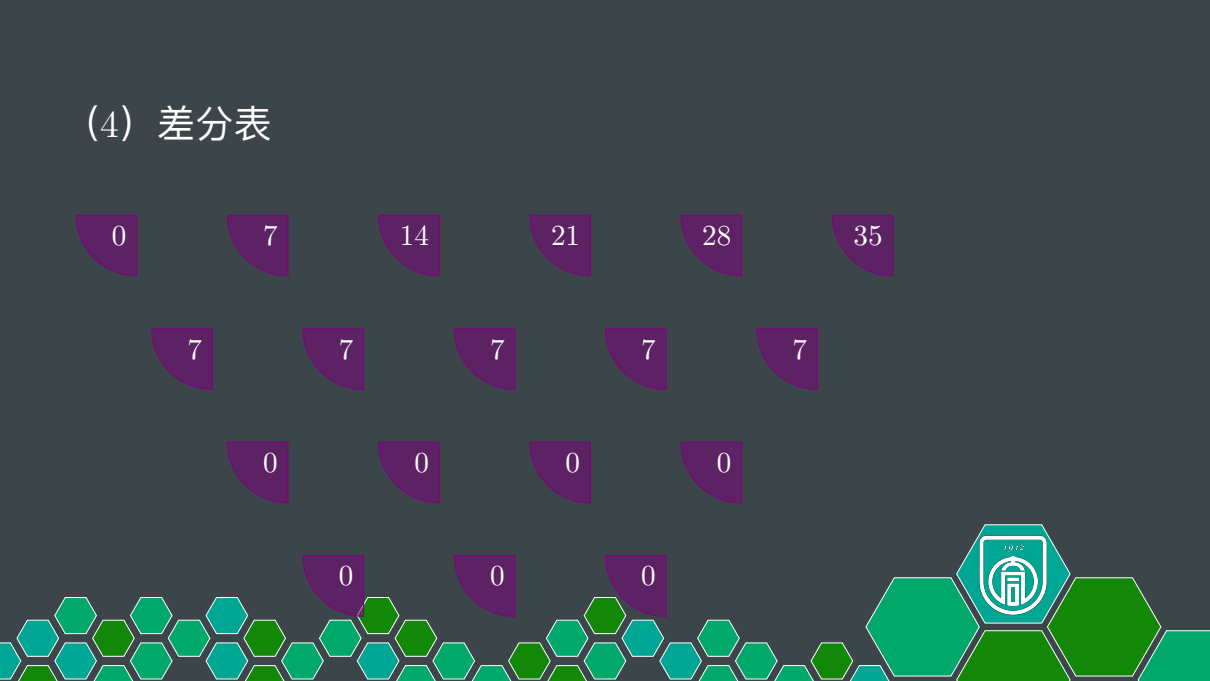


(4) 差分表


0 0 12 36 72 120 0 0 1 3 6 10
0 12 24 36 48 0 1 2 3 4
12 12 12 12 1 1 1 1
0 0 0 0 0 0
×1
分拆
×12

(4) 差分表

0	7	14	21	28	35
7	7	7	7	7	
0	0	0	0		
0	0	0			



(4) 差分表

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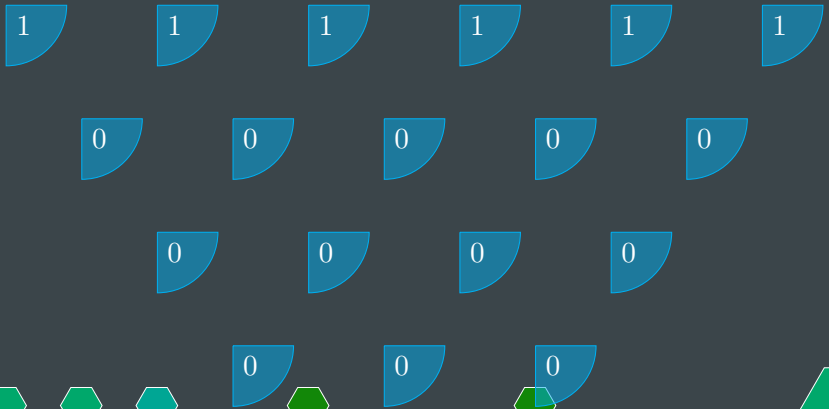
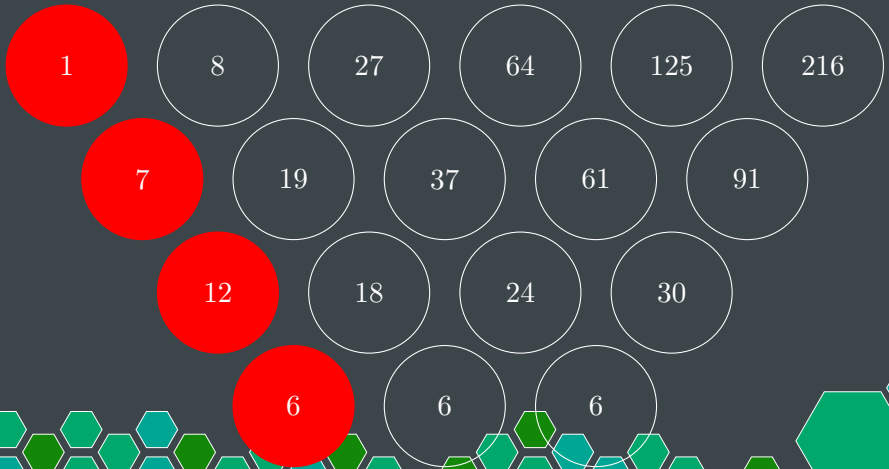
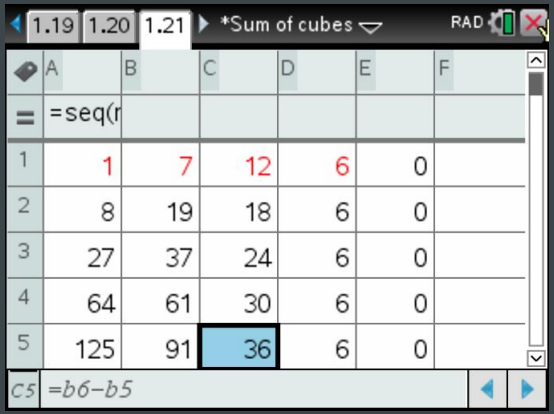
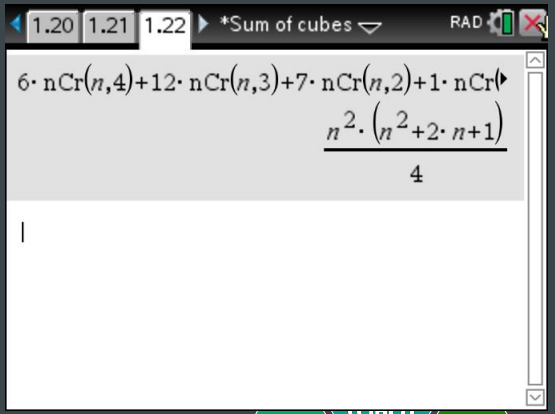


(4) 差分表

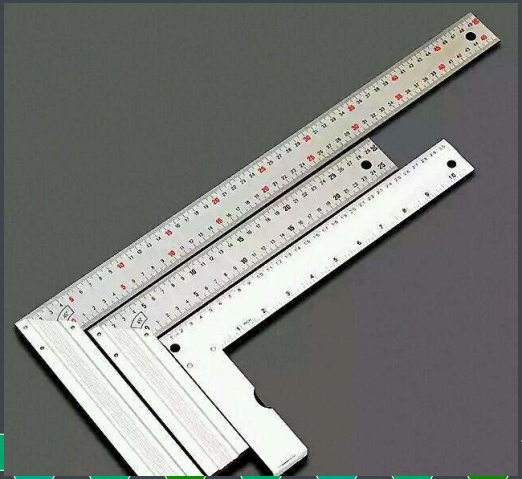


(4) 差分表

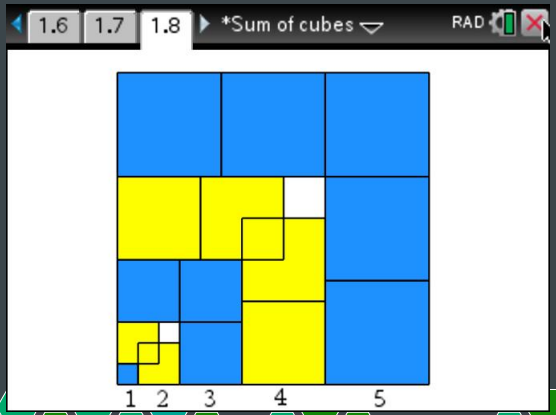

1.19 1.20 1.21 *Sum of cubes RAD
A B C D E F
=seq(r
1 1 7 12 6 0
2 8 19 18 6 0
3 27 37 24 6 0
4 64 61 30 6 0
5 125 91 36 6 0
c5 =b6−b5

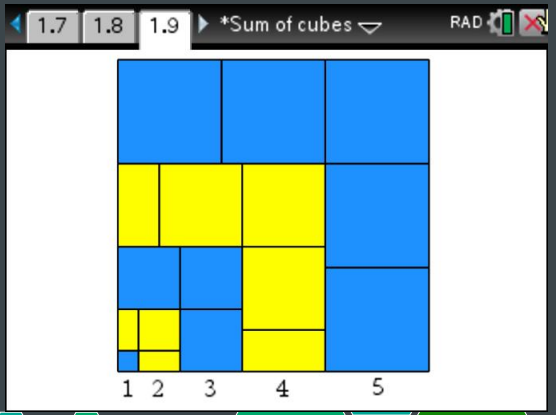

1.20 1.21 1.22 *Sum of cubes RAD
6· nCr(n,4)+12· nCr(n,3)+7· nCr(n,2)+1· nCr
n^2·(n^2+2·n+1)/4

2．在图形拼接中探究摸索

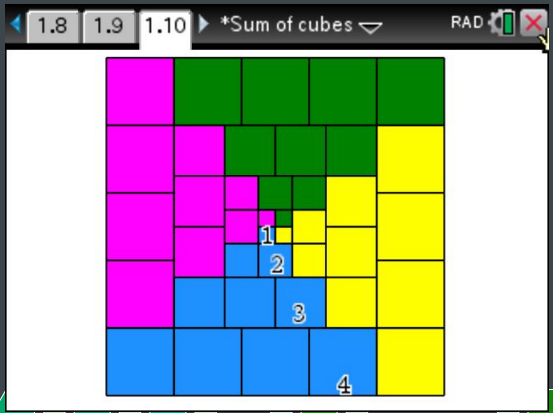


(5) 角尺拼图一、二



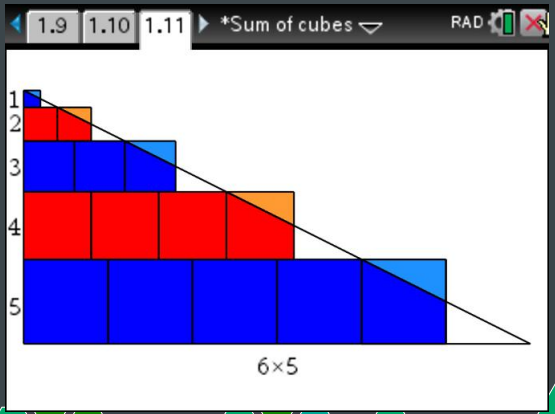


（7）旋转拼图

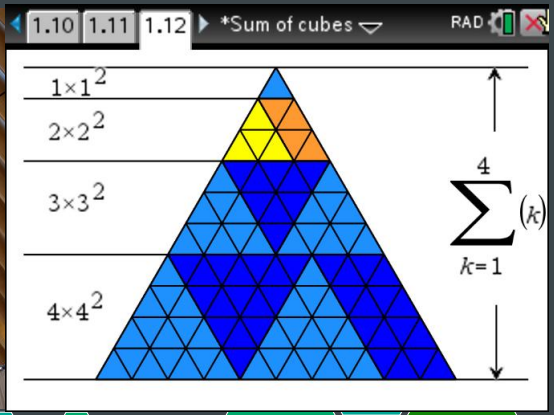


$$\sum_{k=1}^{n} k^3 = \frac{(n\,(n+1))^2}{4}$$

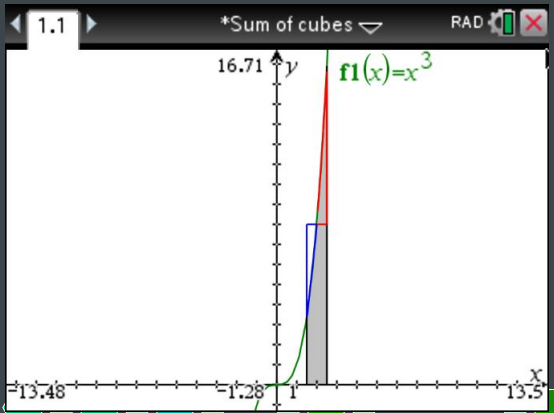
(8)（割补后）三角形拼图



(9) 等边三角形拼图



3．借助技术实现别样想法



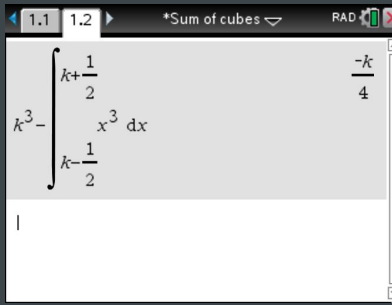
多出一块

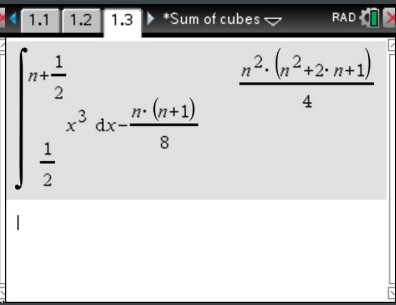
$$\int_{k}^{k+\frac{1}{2}} x^3 dx - \frac{k^3}{2}$$

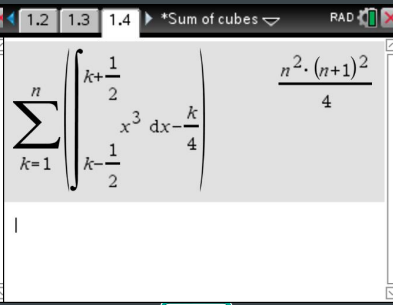
少掉一块

$$\frac{k^3}{2} - \int_{k-\frac{1}{2}}^{k} x^3 dx$$

（10）积分思想



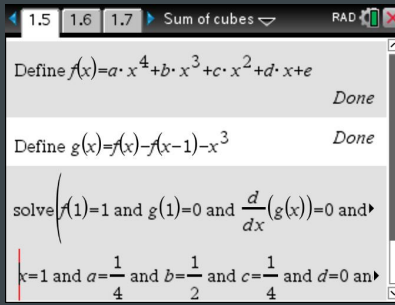




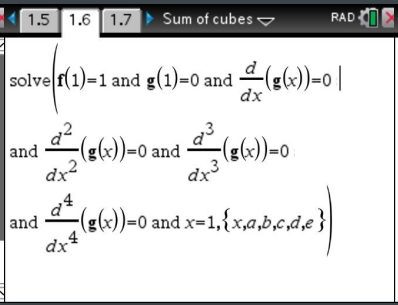
曲边梯形相较于矩形面积多了 $\frac{k}{4}$

TI-Nspire 的计算机代数系统（CAS）

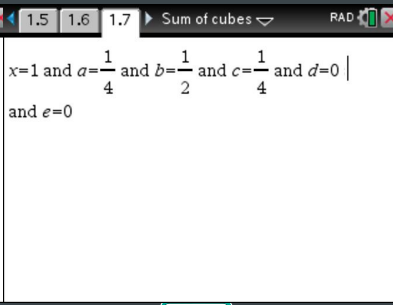
（11）导数思想



适当定义新函数



导数功能与求解方程组功能



4. 大胆尝试技术验证

- 运用累加化归
- 运用 Abel 变换化归
- 运用二项式定理化归
- 运用组合数性质二化归
- 裂项相消

化归

$$\sum_{k=1}^{n} k^2 \frac{(k+1)^2-(k-1)^2}{4}$$

$$\sum_{k=1}^{n} k(k+1)(k+2)$$

$$\sum_{k=1}^{n} k(k+1)$$

$$\sum_{k=1}^{n} (k-1)k(k+1)$$

$$\sum_{k=1}^{n} k$$

$$\sum_{k=1}^{n} k^2$$

$$\sum_{k=1}^{n} k^3$$

参考文献

[1] 徐希来. 中学数学课堂教学中提高记忆效能策略的研究 [D]. 上海: 华东师范大学,2018:67-71.

[2] 张礼恩. 对正整数立方和公式推导的赏析 [J]. 上海中学数学.2012,(11):43-45.

Thank you!

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