



# 基于图形计算器

开展对正整数立方和的探究性学习

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# 目录

## 1 问题背景

## 2 问题解决

### ■ 1. 在数的阵列队形中摸索规律

- (1) 三角形数阵
- (2) 正方形数阵
- (3) 矩形数阵
- (4) 差分表

### ■ 2. 在图形拼接中探究摸索

- (5) 角尺拼图一
- (6) 角尺拼图二

### ■ (7) 旋转拼图

### ■ (8) (割补后) 三角形拼图

### ■ (9) 等边三角形拼图

### ■ 3. 借助技术实现别样想法

- (10) 积分思想
- (11) 导数思想

### ■ 4. 大胆尝试技术验证

## 3 参考文献

## 4 致谢



# 1. 在数的阵列队形中摸索规律

$1^3$

$2^3$

$3^3$

$4^3$



## 1. 在数的阵列队形中摸索规律

$$1^3 \longrightarrow 1 \times 1$$

$$2^3 \longrightarrow 2 \times 4$$

$$3^3 \longrightarrow 3 \times 9$$

$$4^3 \longrightarrow 4 \times 16$$



# 1. 在数的阵列队形中摸索规律

$1^3 \rightarrow 1 \times 1$

1

$2^3 \rightarrow 2 \times 4$

3

5

$3^3 \rightarrow 3 \times 9$

7

9

11

$4^3 \rightarrow 4 \times 16$

13

15

17

19



# (1) 三角形数阵

1.12 1.13 1.14 \*Sum of cubes RAD

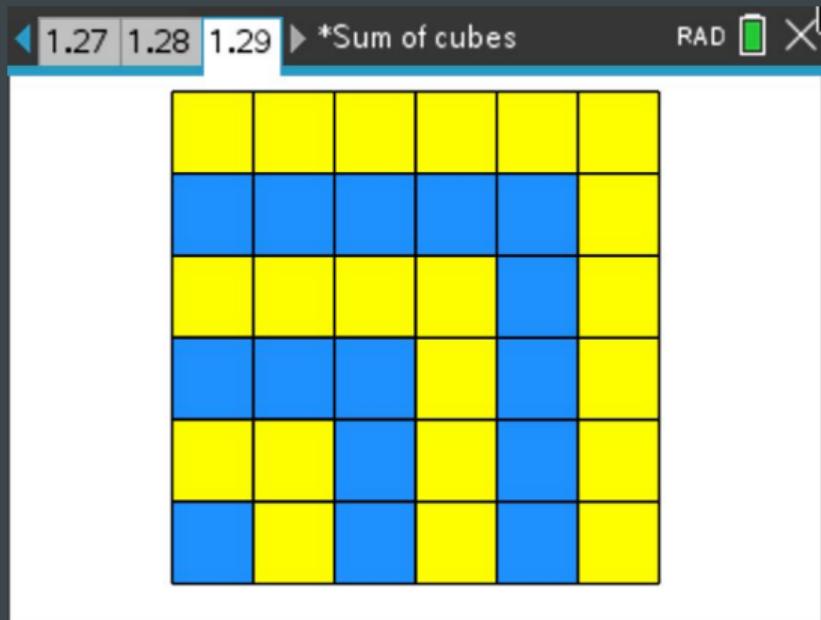
	A	B	C	D	E	F	G
=	=seq(n						
1	1	1					
2	8	3	5				
3	27	7	9	11			
4	64	13	15	17	19		
5	125	21	23	25	27	29	

A cubes:=seq( $n^3, n, 1, 10$ )

1.12 1.13 1.14 \*Sum of cubes RAD

$$\sum_{i=1}^n \left( \sum_{j=1}^i \left( 2 \cdot \frac{i \cdot (i-1)}{2} - 1 + 2 \cdot j \right) \right)$$
$$\frac{n^2 \cdot (n+1)^2}{4}$$

# (1) 三角形数阵



A screenshot of a software interface showing a mathematical formula for the sum of squares. The formula is  $\left(\sum_{k=1}^n k^2\right) = \frac{n^2 \cdot (n+1)^2}{4}$ . The interface includes navigation buttons and a title bar.

从 1 开始的连续  $\frac{n(n+1)}{2}$  个奇数的和

## (2) 正方形数阵

1.15 1.16 1.17 \*Sum of cubes RAD

	A	B	C	D	E	F
=						
1	1	2	3	4	5	
2	2	4	6	8	10	
3	3	6	9	12	15	
4	4	8	12	16	20	
5	5	10	15	20	25	

F6

1.16 1.17 1.18 \*Sum of cubes RAD

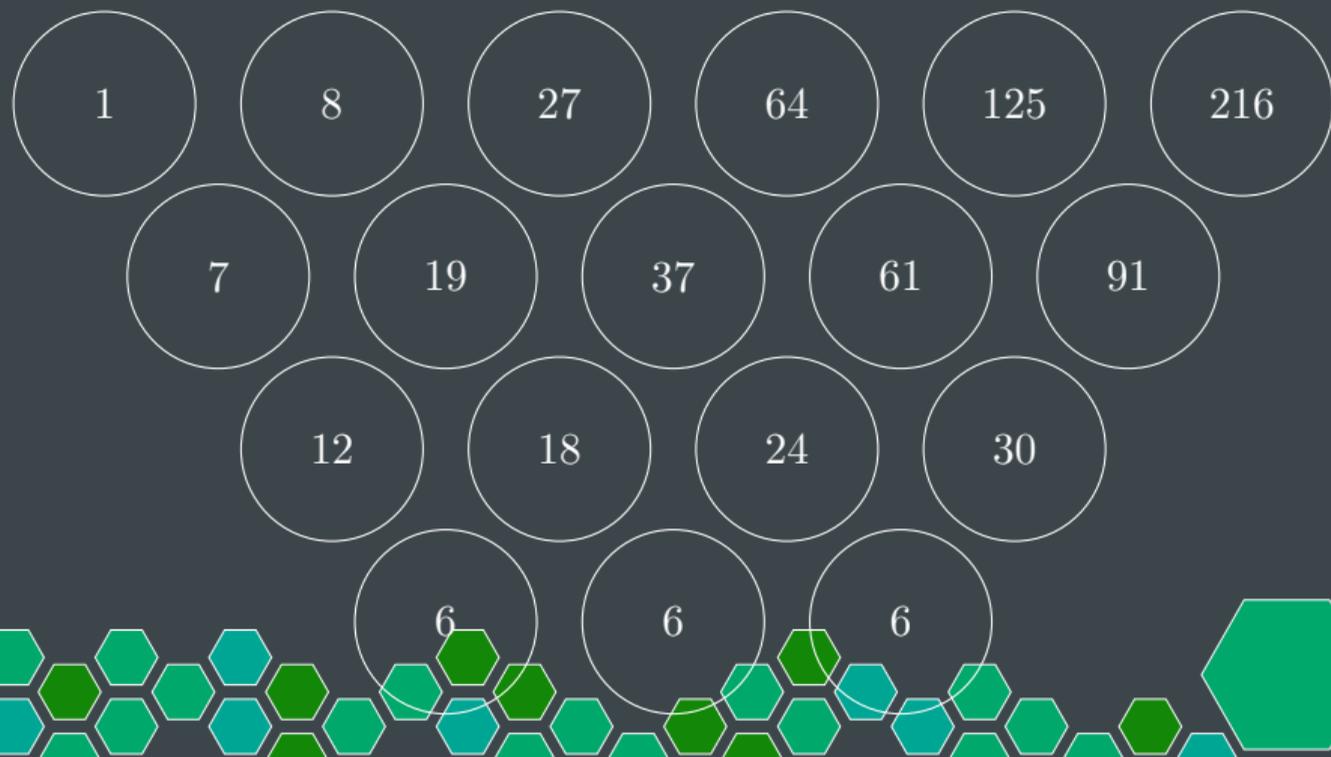
$$\sum_{i=1}^n \left( \sum_{j=1}^n (i \cdot j) \right) = \frac{n^2 \cdot (n+1)^2}{4}$$

### (3) 矩形数阵

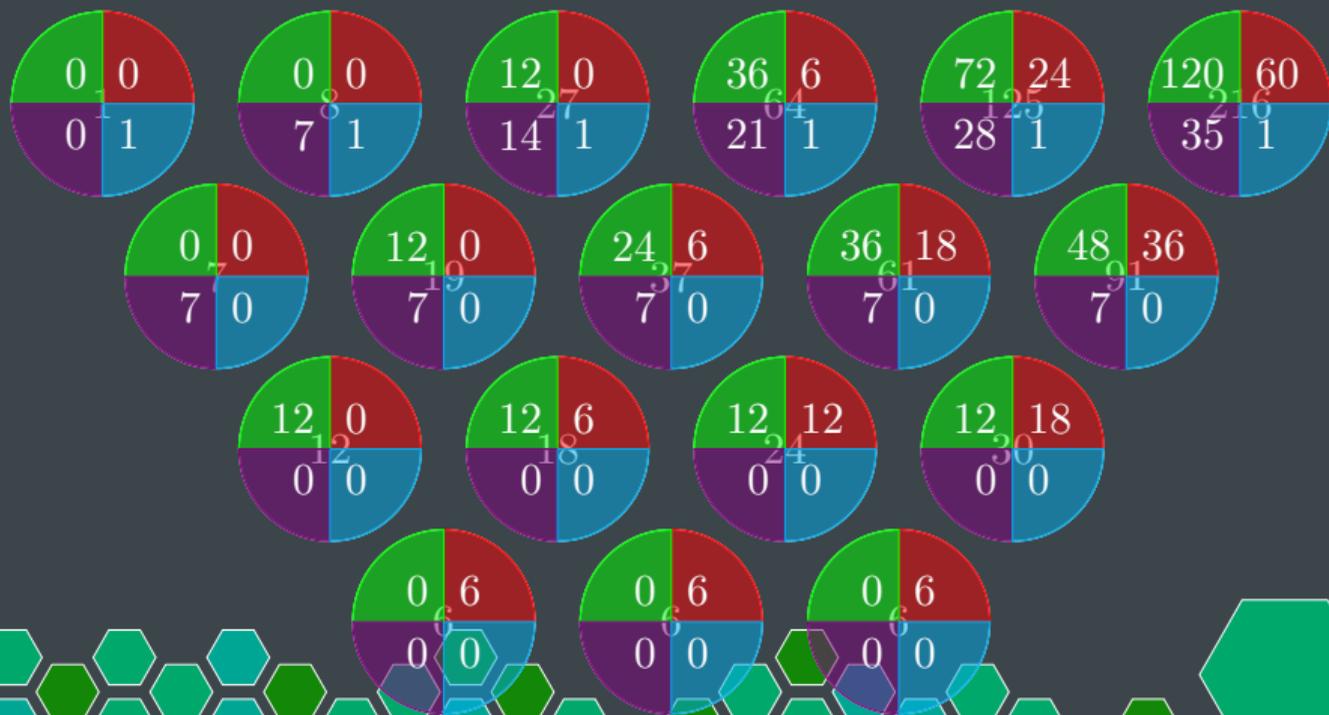
	A	B	C	D	E	F	G
=							
1	1	1	1	1	1	1	
2	4	4	4	4	4	4	
3	9	9	9	9	9	9	
4	16	16	16	16	16	16	
5	25	25	25	25	25	25	

$$(n+1) \cdot \sum_{k=1}^n (k^2) - \sum_{i=1}^n \left( \sum_{j=1}^i (j^2) \right)$$
$$\frac{n^2 \cdot (n+1)^2}{4}$$

#### (4) 差分表



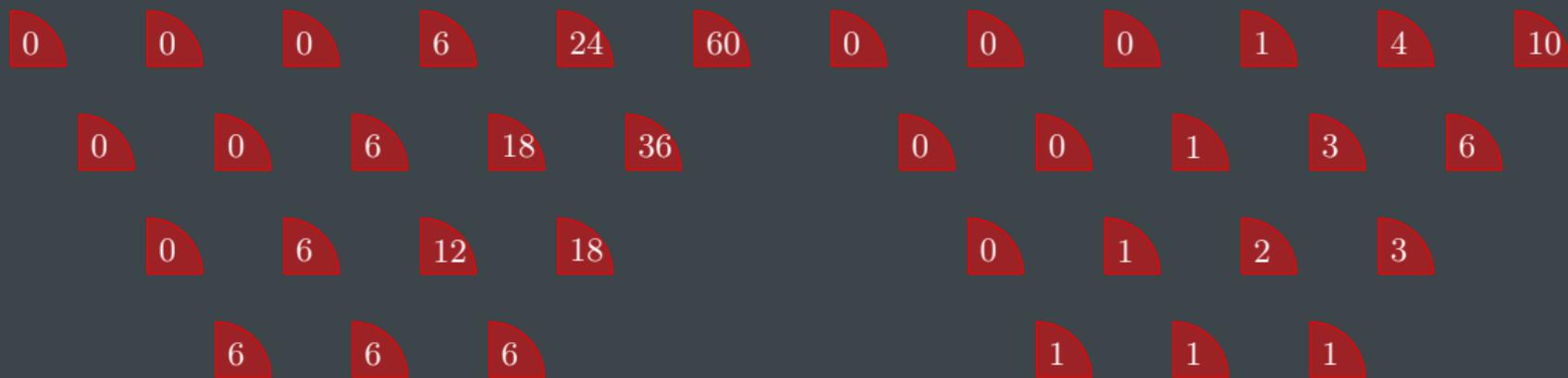
#### (4) 差分表



#### (4) 差分表



## (4) 差分表



$\times 1$

分拆

$\times 6$



#### (4) 差分表



## (4) 差分表



$\times 1$   $\xrightarrow{\text{分拆}}$   $\times 12$



## (4) 差分表

0

7

14

21

28

35

7

7

7

7

7

0

0

0

0

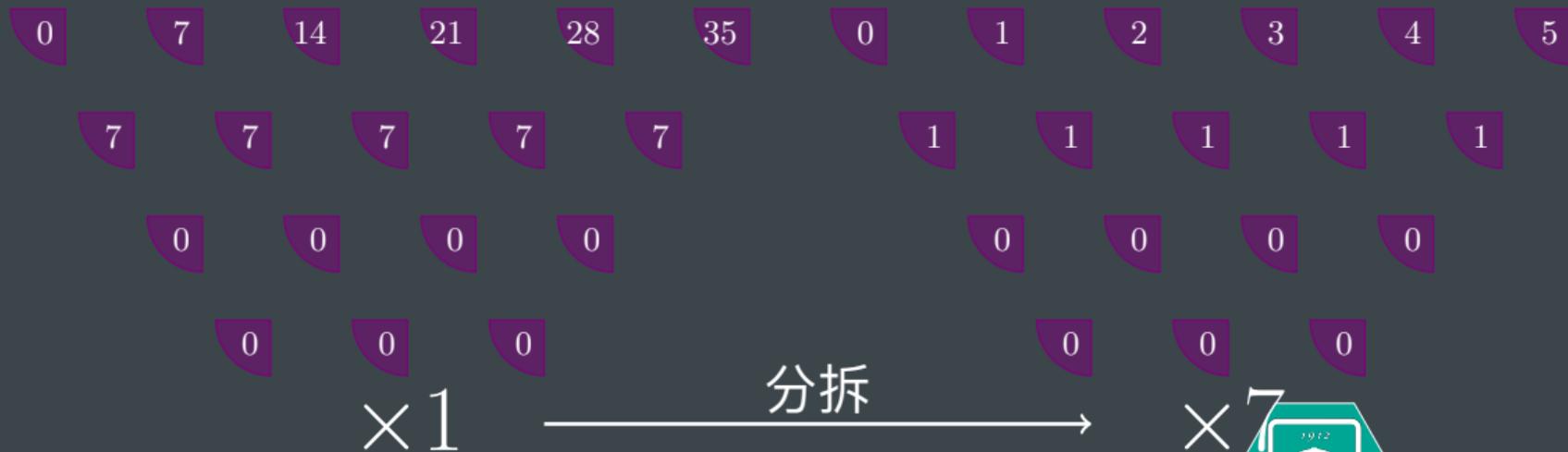
0

0

0



## (4) 差分表

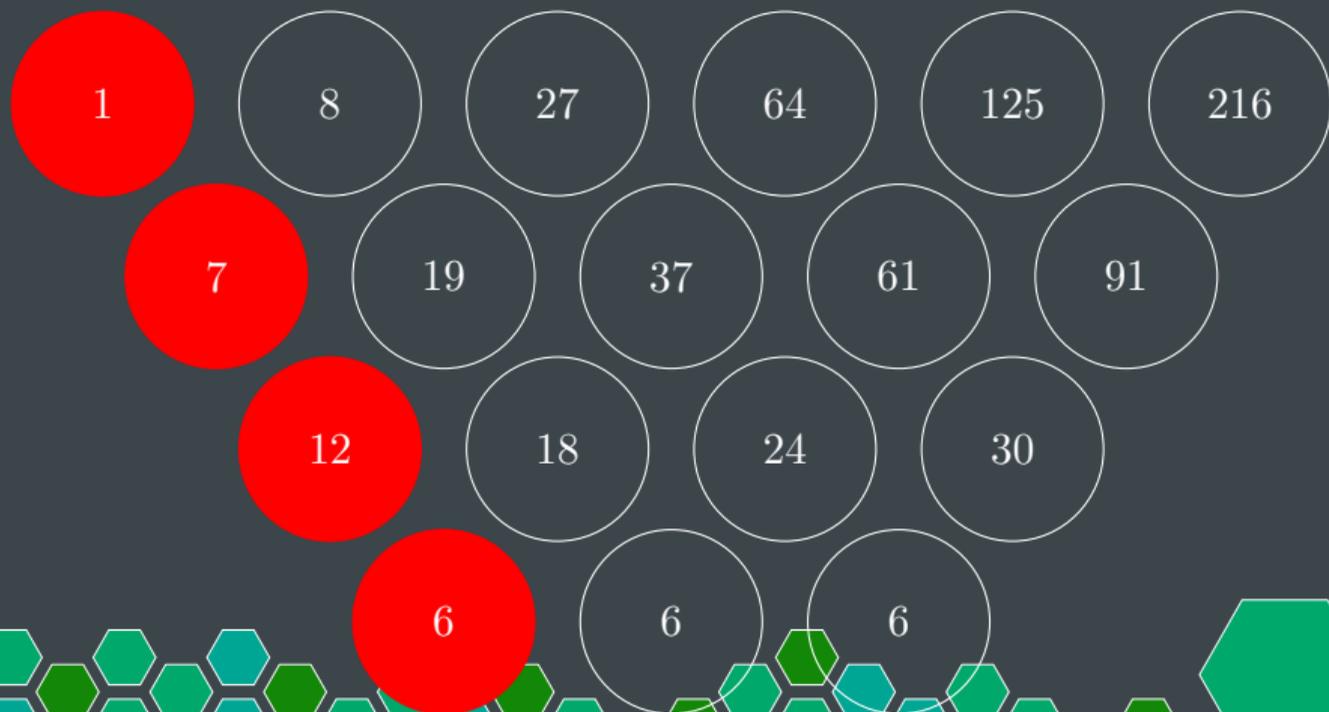


#### (4) 差分表

1	1	1	1	1	1
	0	0	0	0	0
		0	0	0	
			0	0	
				0	



#### (4) 差分表



#### (4) 差分表

1.19 1.20 1.21 \*Sum of cubes RAD

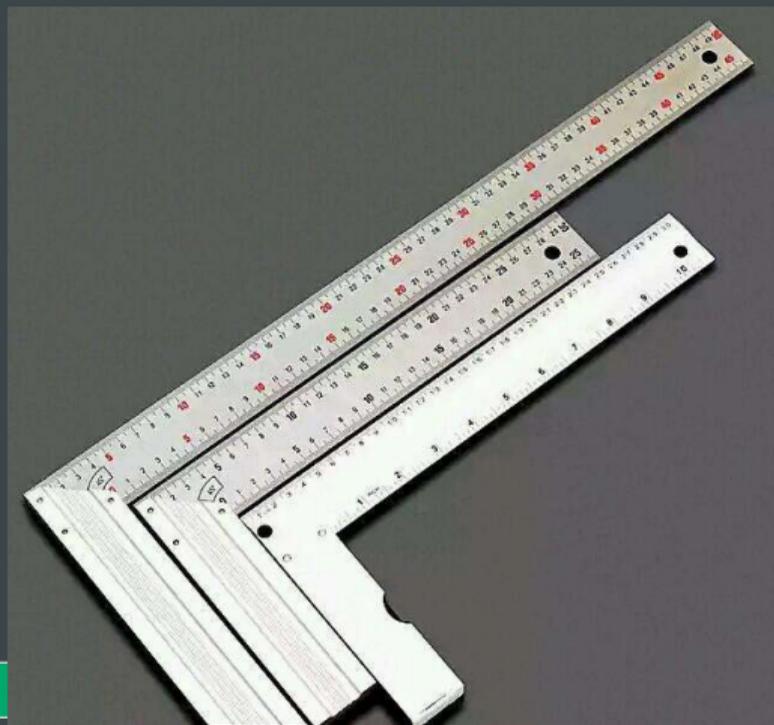
	A	B	C	D	E	F
=	=seq(r					
1	1	7	12	6	0	
2	8	19	18	6	0	
3	27	37	24	6	0	
4	64	61	30	6	0	
5	125	91	36	6	0	

C5 =b6-b5

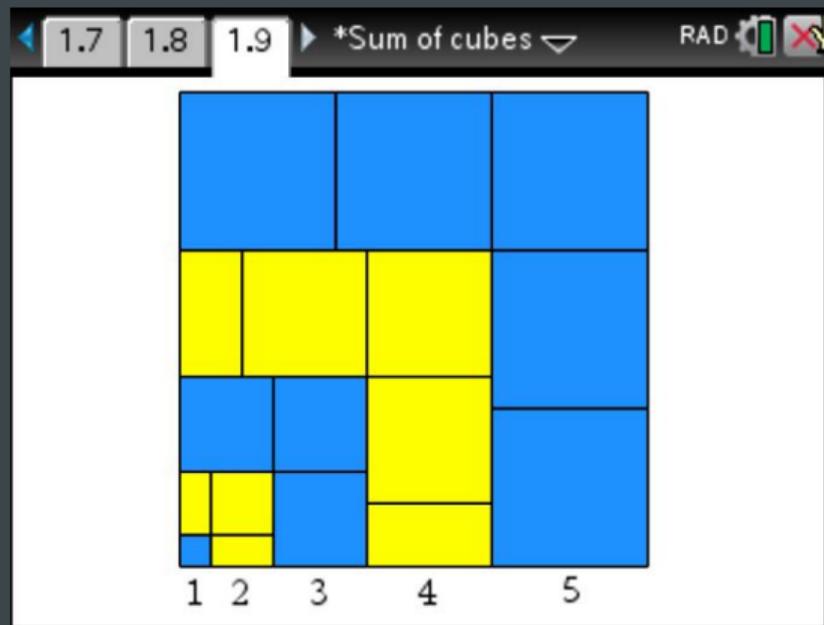
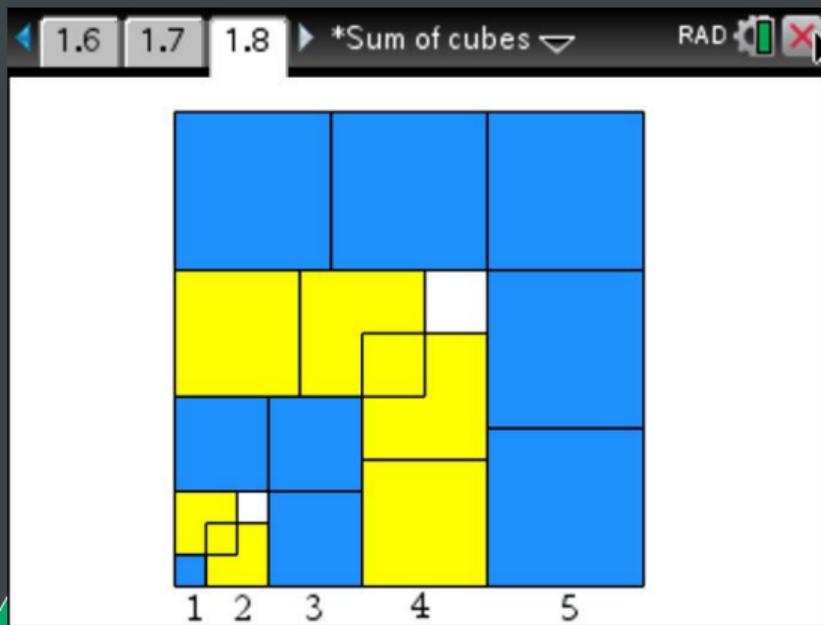
1.20 1.21 1.22 \*Sum of cubes RAD

$$\frac{6 \cdot nCr(n,4) + 12 \cdot nCr(n,3) + 7 \cdot nCr(n,2) + 1 \cdot nCr(n,1)}{4}$$

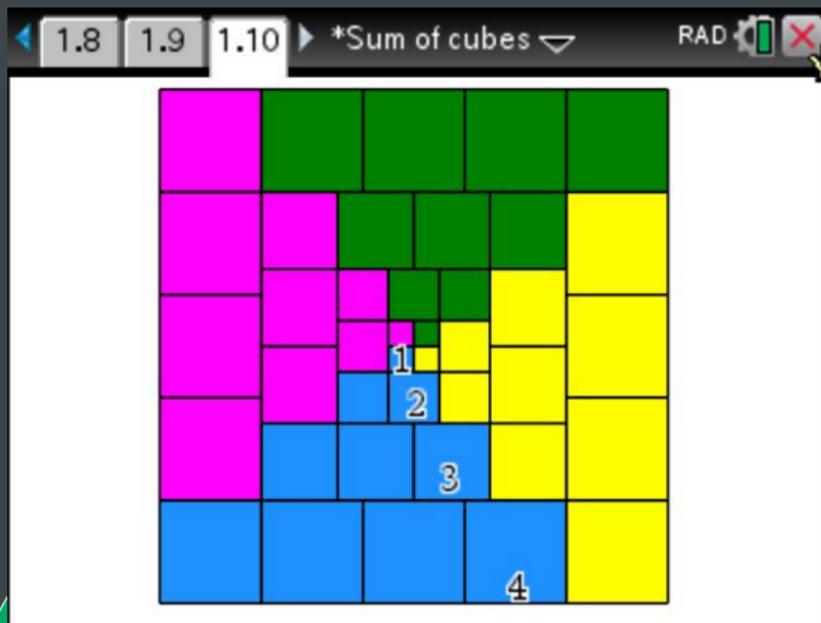
## 2. 在图形拼接中探究摸索



## (5) 角尺拼图一、二



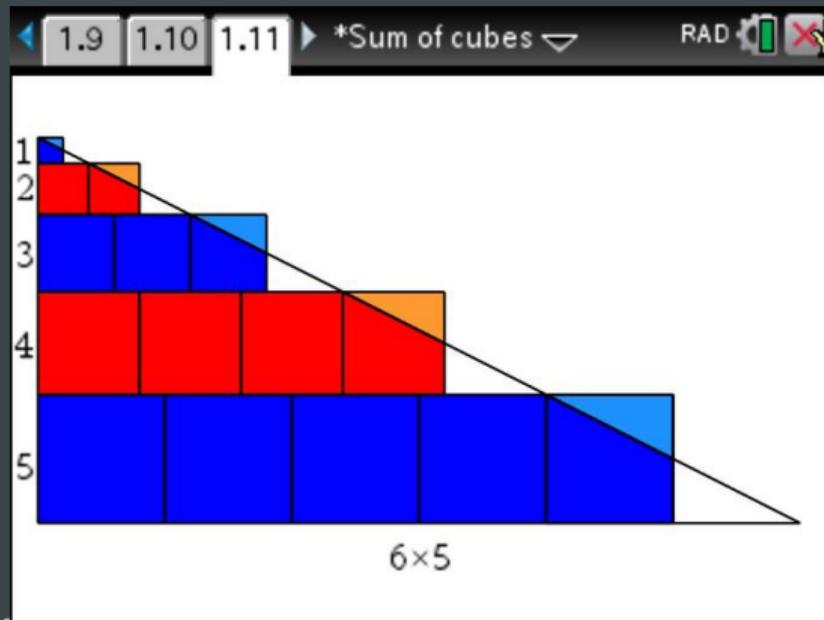
## (7) 旋转拼图



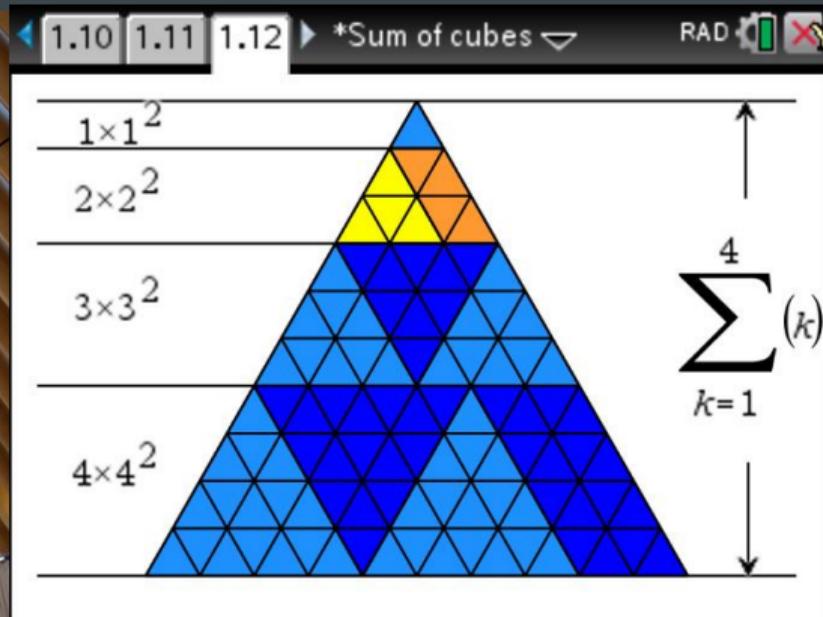
$$\sum_{k=1}^n k^3 = \frac{(n(n+1))^2}{4}$$



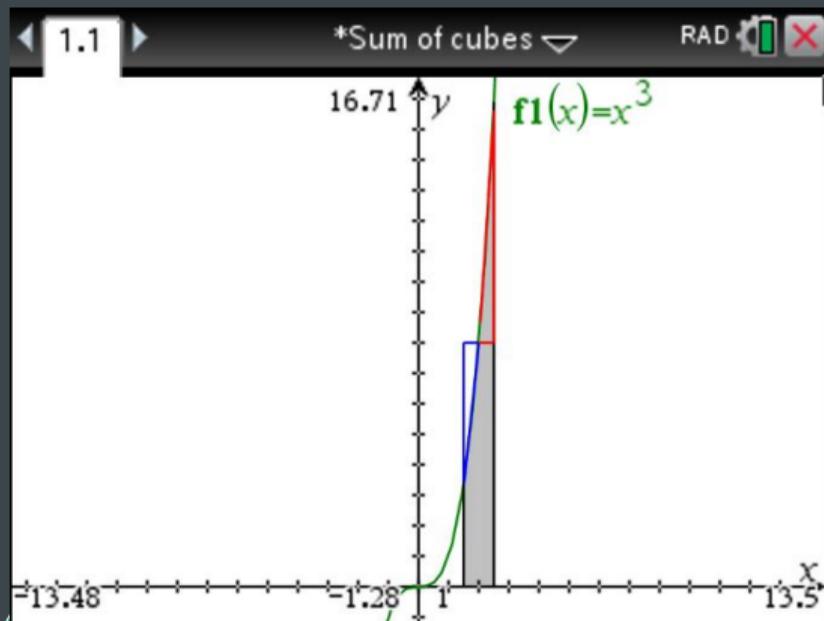
## (8) (割补后) 三角形拼图



## (9) 等边三角形拼图



### 3. 借助技术实现别样想法



多出一块

$$\int_k^{k+\frac{1}{2}} x^3 dx - \frac{k^3}{2}$$

少掉一块

$$\frac{k^3}{2} - \int_{k-\frac{1}{2}}^k x^3 dx$$



## (10) 积分思想

The image shows three sequential TI-Nspire CAS calculator screens illustrating the integral approach to finding the area of a trapezoid. The first screen shows the integral of  $x^3$  from  $k - \frac{1}{2}$  to  $k + \frac{1}{2}$ , resulting in  $\frac{-k}{4}$ . The second screen shows the integral from  $\frac{1}{2}$  to  $n + \frac{1}{2}$ , resulting in  $\frac{n^2 \cdot (n^2 + 2 \cdot n + 1)}{4} - \frac{n \cdot (n + 1)}{8}$ . The third screen shows the summation of these integrals from  $k = 1$  to  $n$ , resulting in  $\frac{n^2 \cdot (n + 1)^2}{4}$ .

$$k^3 - \int_{k - \frac{1}{2}}^{k + \frac{1}{2}} x^3 dx = \frac{-k}{4}$$
$$\int_{\frac{1}{2}}^{n + \frac{1}{2}} x^3 dx = \frac{n^2 \cdot (n^2 + 2 \cdot n + 1)}{4} - \frac{n \cdot (n + 1)}{8}$$
$$\sum_{k=1}^n \left( \int_{k - \frac{1}{2}}^{k + \frac{1}{2}} x^3 dx - \frac{k}{4} \right) = \frac{n^2 \cdot (n + 1)^2}{4}$$

曲边梯形相较于矩形面积多了  $\frac{1}{4}$

TI-Nspire 的计算机代数系统 (CAS)



## (11) 导数思想

The image shows three sequential screenshots of a TI-84 Plus calculator interface, illustrating the process of defining a function and solving for its parameters using derivative functions.

**Left Screenshot:** Shows the definition of a function  $f(x) = a \cdot x^4 + b \cdot x^3 + c \cdot x^2 + d \cdot x + e$  and the definition of a new function  $g(x) = f(x) - f(x-1) - x^3$ . The calculator prompts for the values of  $a, b, c, d,$  and  $e$ .

**Middle Screenshot:** Shows the use of the  $\text{solve}$  function to solve the system of equations:  $f(1) = 1$  and  $g(1) = 0$  and  $\frac{d}{dx}(g(x)) = 0$  and  $\frac{d^2}{dx^2}(g(x)) = 0$  and  $\frac{d^3}{dx^3}(g(x)) = 0$  and  $\frac{d^4}{dx^4}(g(x)) = 0$  and  $x = 1, \{x, a, b, c, d, e\}$ .

**Right Screenshot:** Shows the solution:  $x = 1$  and  $a = \frac{1}{4}$  and  $b = \frac{1}{2}$  and  $c = \frac{1}{4}$  and  $d = 0$  and  $e = 0$ .

适当定义新函数

导数功能与求解方程组功能



## 4. 大胆尝试技术验证

- 运用累加化归
- 运用 Abel 变换化归
- 运用二项式定理化归
- 运用组合数性质二化归
- 裂项相消



# 化归

$$\sum_{k=1}^n k$$

$$\sum_{k=1}^n k(k+1)$$

$$\sum_{k=1}^n k^2$$

$$\sum_{k=1}^n (k-1)k(k+1)$$

$$\sum_{k=1}^n k(k+1)(k+2)$$

$$\sum_{k=1}^n k^3$$

$$\sum_{k=1}^n k^2 \frac{(k+1)^2 - (k-1)^2}{4}$$



## 参考文献

- [1] 徐希来. 中学数学课堂教学中提高记忆效能策略的研究 [D]. 上海: 华东师范大学,2018:67-71.
- [2] 张礼恩. 对正整数立方和公式推导的赏析 [J]. 上海中学数学.2012,(11):43-45.



Thank you!

I am reachable at [xuxilai2003@163.com](mailto:xuxilai2003@163.com).

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