Semi-implicit Euler-Maruyama method for stochastic differential equations driven by symmetric α-stable process

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Abstract

The semi-implicit Euler Maruyama method is studied to approximate stochastic differential equations (SDEs) driven by both the Brownian motion and the symmetric α-stable process. The drift coefficient of SDEs under consideration is allowed to grow super-linearly. The strong convergence of the semi-implicit EM is shown with the rate of $(\alpha - \varepsilon)/4$ for any arbitrarily small $\varepsilon > 0$ and $\alpha \in [1, 2)$.

Keywords: semi-implicit Euler-Maruyama method; stochastic differential equations; strong convergence; α-stable process

1 Introduction

This paper is devoted to study the numerical approximation to a class of stochastic differential equations (SDEs) driven by both the Brownian motion, $B(t)$ and the symmetric α-stable process, $L(t)$ of the form

$$\mathrm{d}x(t) = f(x(t))\mathrm{d}t + g(x(t))\mathrm{d}B(t) + \mathrm{d}L(t), \quad 0 \le t \le T, \tag{1.1}$$

with the initial value $x(0) = x_0$.

Janicki, Michna and Weron in [?] studied the Euler Maruyama (EM) method when the coefficients of (??) satisfy the global Lipschitz condition. Pamen and Taguchi in [?] investigated the EM method for the case that $L(t)$ is a truncated α-stable process and the drift coefficient is Hölder continuous. Huang and Liao in [?] extended the results to stochastic functional results when the driven noise is the symmetric α-stable process. To our best knowledge, it seems that few result on the numerical approximation is available for (??) when the drift coefficient is allowed to grow super-linearly. Meanwhile, super-linear

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terms appear in SDE models frequently. To fill in this gap, the main purpose of this paper is to study the numerical approximation to this class of SDEs.

As indicated by Hutzenthaler, Jentzen and Kloeden in [**?**], the EM method fails to convergence when some super-linearity appears in the coefficients. Therefore, we propose the semi-implicit EM method for (**??**) when the drift coefficient may grow super-linearly. The semi-implicit EM method has been extensively investigated for (**??**) when the $L(t)$ is absent, see for example [**?**] and the references therein. The strong convergence rate is proved to be $1/2$ in this case due to that the second moment of $B(t)$ is t. The main difference and difficulty of this paper are brought in by the extra driven process $L(t)$, which is a generalisation of $B(t)$ with the moments dependent on the α. More precisely,

$$\mathbb{E}|L(t)|^q \leq C_1 t^{q/\alpha}, \tag{1.2}$$

for any $q \in [1, \alpha)$. (see Property 1.2.17 at Page 18 of [**?**])

In this paper, we show that the semi-implicit EM method converge strongly to (**??**) with the super-linear drift coefficient and the convergence rate is $(\alpha - \varepsilon)/4$ for any arbitrarily small $\varepsilon > 0$ and $\alpha \in [1, 2)$.

The paper is constructed as follows. Section **??** contains assumptions and useful lemmas. The main result is presented and proved in Section **??**. An example is provided in **??** to illustrate that our main can cover a large class of non-linear SDEs.

2 Mathematical preliminary

Throughout this paper, unless otherwise specified, we use the following notation. If A is a vector or matrix, its transpose is denoted by A^T. For $x \in \mathbb{R}^n$, $|x|$ denotes its Euclidean norm. If A is a matrix, we let $|A| = \sqrt{\text{trace}(A^T A)}$ be its trace norm. C stands for a generic positive real constant.

Let $\left(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t \geq 0}, \mathbb{P}\right)$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e. it is increasing and right continuous while $\mathscr{F}_0$ contains all $\mathbb{P}$-null sets). Let $B(t) = \left(B_1(t), \cdots, B_m(t)\right)^T$ be an m-dimensional Brownian motion defined on the probability space.

In this paper, we consider the following stochastic differential equations driven by symmetric α-stable process

$$\mathrm{d}x(t) = f(x(t))\mathrm{d}t + g(x(t))\mathrm{d}B(t) + \mathrm{d}L(t), \quad 0 \leq t \leq T, \tag{2.1}$$

with the initial value $x(0) = x_0$, where $f : \mathbb{R}^n \mapsto \mathbb{R}^n$ and $g : \mathbb{R}^n \mapsto \mathbb{R}^{n \times m}$ are measurable functions. Here $L(t)$ is a scalar symmetric α-stable process. Throughout this paper, we assume that $\alpha \in (1, 2)$.

In Chapter 1 of [**?**], the authors present four equivalent ways to describe the α-stable process. In this paper, we adopt the following description. A stochastic process $L = \{L(t)\}_{0 \leq t \leq T}$ on $\mathbb{R}^d$ is called a α-stable process on the probability space $\left(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t \geq 0}, \mathbb{P}\right)$ if the following conditions are satisfied:

- $L(0) = 0$, a.s.
- For any $n \in \mathbb{N}$ and $0 \leq t_1 < t_2 < \cdots < t_n \leq T$, the random variables $L(t_0)$, $L(t_1) - L(t_0)$, $L(t_2) - L(t_1)$, $\cdots$ $L(t_n) - L(t_{n-1})$ are independent.
- For any $0 \leq s < t < \infty$, $L(t) - L(s)$ follows $S_\alpha((t-s)^{1/\alpha}, 0, 0))$, where $S_\alpha(\sigma, \beta, \mu)$ is a four-parameter stable distribution.

It should be mentioned that such a description makes numerical simulations quite straightforward. The symmetric α-stable process belongs to the family of Lévy process. We refer the readers to [?] for the detailed introduction to Lévy processes driven SDEs.

Now, we impose the following assumptions on the drift and diffusion coefficients.

(**A1**) Assume that there exists a constant $K_1 > 0$ such that for any $q \geq 2$ and $x \in \mathbb{R}^n$

$$x^T f(x) + \frac{q-1}{2}|g(x)|^2 \leq K_1(1 + |x|^2). \tag{2.2}$$

(**A2**) Assume that there exist constants $K_2 > 0$ and $\gamma > 0$ such that for any $x, y \in \mathbb{R}^n$

$$|f(x) - f(y)|^2 \leq K_2(1 + |x|^\gamma + |y|^\gamma)|x - y|^2, \tag{2.3}$$

and

$$|g(x) - g(y)|^2 \leq K_2|x - y|^2. \tag{2.4}$$

(**A3**) Assume that there exists a constant $K_3 > 0$ such that for any $x, y \in \mathbb{R}^n$

$$(x - y)^T(f(x) - f(y)) \leq K_3|x - y|^2. \tag{2.5}$$

For some given time step $\Delta \in (0, 1)$ and the terminal time T, define $N = T/\Delta$. The semi-implicit Euler-Maruyama method for (**??**) is defined by

$$y_{i+1} = y_i + f(y_{i+1})\Delta + g(y_i)\Delta B_i + \Delta L_i, \tag{2.6}$$

with $y(0) = x_0$, where ΔB_i is the Brownian increment following the normal distribution with the mean 0 and the variance Δ and $\Delta L_i = L(t_{i+1}) - L(t_i)$ follows the stable distribution $S_\alpha(\Delta^{1/\alpha}, 0, 0))$ for $i = 1, 2, \cdots, N$. Thus, y_i is the approximation to $x(i\Delta)$ for $i = 1, 2, \cdots, N$.

We also define the piecewise continuous numerical solution by

$$y(t) = y_i, \quad \text{for } t \in [i\Delta, (i+1)\Delta),\ i = 1, 2, \cdots. \tag{2.7}$$

The following lemmas are needed for the proof of our main result.

Lemma 2.1. *Suppose that (**A1**) holds, then the solution to (**??**) satisfies*

$$\mathbb{E}|x(t)|^q \leq C. \tag{2.8}$$

Proof: By the Itô formula, we have that for any $0 \le t \le T$,

$$d|x(t)|^q = |x(0)|^q + \int_0^t \left[q|x(s)|^{q-2} x^T f(x(s)) + \frac{q(q-1)}{2} |x(s)|^{q-2} |g(x(s))|^2 \right] ds + M(t),$$

where $M(t)$ is a martingale with $\mathbb{E}M(t) = 0$. By (**??**), we can see that

$$\begin{aligned}
\mathbb{E}|x(t)|^q &\le \mathbb{E}|x_0|^q + qK_1 \mathbb{E} \int_0^t |x(s)|^{q-2}(1 + |x(s)|^2)\, ds \\
&\le \mathbb{E}|x_0|^q + qK_1 \mathbb{E} \int_0^t (1 + 2|x(s)|^q)\, ds \\
&\le \mathbb{E}|x_0|^q + qK_1 t + 2qK_1 \mathbb{E} \int_0^t |x(s)|^q\, ds.
\end{aligned}$$

An application of Gronwall's inequality yields the required assertion. □

Lemma 2.2. *Suppose that (**A2**) holds, then for any $1 < p < \alpha$ and $|t - s| < 1$*

$$\mathbb{E}|x(t) - x(s)|^p \le C|t - s|^{\frac{p}{2}}. \tag{2.9}$$

Proof: For any $0 \le s < t$, we derive from (**??**) that

$$x(t) - x(s) = \int_s^t f(x(r))\, dr + \int_s^t g(x(r))\, dB(r) + (L(t) - L(s)).$$

Taking pth moment on both sides, by the elementary inequality $|u + v + w|^p \le 3^{p-1}(|u|^p + |v|^p + |w|^p)$ for any $u, v, w \in \mathbb{R}^n$, Hölder's inequality, Itô isometry and (**??**) we have

$$\begin{aligned}
\mathbb{E}|x(t) - x(s)|^p &= \mathbb{E} \left| \int_s^t f(x(r))\, dr + \int_s^t g(x(r))\, dB(r) + (L(t) - L(s)) \right|^p \\
&\le 3^{p-1} \mathbb{E} \left| \int_s^t f(x(r))\, dr \right|^p + 3^{p-1} \mathbb{E} \left| \int_s^t g(x(r))\, dB(r) \right|^p + 3^{p-1} \mathbb{E} \left| L(t) - L(s) \right|^p \\
&\le C \left(|t - s|^p + |t - s|^{\frac{p}{2}} + |t - s|^{\frac{p}{\alpha}} \right) \\
&\le C|t - s|^{\frac{p}{2}}.
\end{aligned}$$

Therefore, the proof is completed. □

3 Main result

Theorem 3.1. *Suppose that (**A1**), (**A2**) and (**A3**) hold, then for any $\varepsilon > 0$ the semi-implicit Euler-Maruyama method (**??**) is convergent to (**??**) with the rate of $(\alpha - \varepsilon)/4$, i.e.*

$$\mathbb{E}|x(t) - y(t)|^2 \le C\Delta^{\frac{\alpha - \varepsilon}{2}}, \quad \textit{for any } t \in [0, T]. \tag{3.1}$$

Proof: From (**??**) and (**??**), we can see that for $i = 1, 2, \cdots, N$

$$x(t_{i+1}) = x(t_i) + \int_{t_i}^{t_{i+1}} f(x(s))\, ds + \int_{t_i}^{t_{i+1}} g(x(s))\, dB(s) + (L(t_{i+1}) - L(t_i))$$

and

$$y_{i+1} = y_i + \int_{t_i}^{t_{i+1}} f(y_{i+1})\, ds + \int_{t_i}^{t_{i+1}} g(y_i)\, dB(s) + (L(t_{i+1}) - L(t_i)).$$

So, we have

$$x(t_{i+1}) - y_{i+1} = (x(t_i) - y_i) + \int_{t_i}^{t_{i+1}} (f(x(s)) - f(y_{i+1}))\, ds + \int_{t_i}^{t_{i+1}} (g(x(s)) - g(y_i))\, dB(s).$$

Taking square on both sides yields

$$|x(t_{i+1}) - y_{i+1}|^2 := A_1 + A_2,$$

where

$$A_1 = \int_{t_i}^{t_{i+1}} (x(t_{i+1}) - y_{i+1})^T (f(x(s)) - f(y_{i+1}))\, ds$$

and

$$A_2 = (x(t_{i+1}) - y_{i+1})^T \left((x(t_i) - y_i) + \int_{t_i}^{t_{i+1}} (g(x(s)) - g(y_i))\, dB(s) \right).$$

To estimate A_1, we need rewrite the integrand of A_1 into two parts

$$\begin{aligned} &(x(t_{i+1}) - y_{i+1})^T (f(x(s)) - f(y_{i+1})) \\ &= (x(t_{i+1}) - y_{i+1})^T (f(x(t_{i+1})) - f(y_{i+1})) + (x(t_{i+1}) - y_{i+1})^T (f(x(s)) - f(x(t_{i+1}))) \\ &:= A_{11} + A_{12}. \end{aligned}$$

Using (**??**), we can obtain

$$A_{11} \le K_3 |x(t_{i+1}) - y_{i+1}|^2.$$

Applying the elementary inequality $u^T v \le \frac{1}{2}|u|^2 + \frac{1}{2}|v|^2$ for $u, v \in \mathbb{R}^n$ and (**??**), we have

$$\begin{aligned} A_{12} &\le \frac{1}{2}|x(t_{i+1}) - y_{i+1}|^2 + \frac{1}{2}|f(x(s)) - f(x(t_{i+1}))|^2 \\ &\le \frac{1}{2}|x(t_{i+1}) - y_{i+1}|^2 + \frac{K_2}{2}(1 + |x(s)|^\gamma + |x(t_{i+1})|^\gamma)|x(s) - x(t_{i+1})|^2. \end{aligned}$$

Therefore, we have

$$A_1 \le \int_{t_i}^{t_{i+1}} \left[(K_3 + \frac{1}{2})|x(t_{i+1}) - y_{i+1}|^2 + \frac{K_2}{2}(1 + |x(s)|^\gamma + |x(t_{i+1})|^\gamma)|x(s) - x(t_{i+1})|^2 \right] ds. \tag{3.2}$$

Using the elementary inequality, we can get, for any $\varepsilon > 0$,

$$\begin{aligned}
&(1+|x(s)|^\gamma+|x(t_{i+1})|^\gamma)|x(s)-x(t_{i+1})|^{2-\alpha+\varepsilon}\\
\le\ &(1+|x(s)|^\gamma+|x(t_{i+1})|^\gamma)(|x(s)|^{2-\alpha+\varepsilon}+|x(t_{i+1})|^{2-\alpha+\varepsilon})\\
\le\ &3(|x(s)|^{2+\gamma}+|x(t_{i+1})|^{2+\gamma}).
\end{aligned}$$

We choose $a = (2\alpha-\varepsilon)/(2\alpha-2\varepsilon)$, then we have $(2+\gamma)\frac{a}{a-1} \ge 2$ and $1 < a(\alpha-\varepsilon) < \alpha$. By Hölder's inequality, (**??**) and (**??**),

$$\begin{aligned}
&\mathbb{E}((1+|x(s)|^\gamma+|x(t_{i+1})|^\gamma)|x(s)-x(t_{i+1})|^2)\\
=\ &\mathbb{E}\left[((1+|x(s)|^\gamma+|x(t_{i+1})|^\gamma)|x(s)-x(t_{i+1})|^{2-\alpha+\varepsilon})|x(s)-x(t_{i+1})|^{\alpha-\varepsilon}\right]\\
\le\ &3\mathbb{E}\left[(|x(s)|^{2+\gamma}+|x(t_{i+1})|^{2+\gamma})|x(s)-x(t_{i+1})|^{\alpha-\varepsilon}\right]\\
\le\ &3\left(\mathbb{E}(|x(s)|^{2+\gamma}+|x(t_{i+1})|^{2+\gamma})^{\frac{a}{a-1}}\right)^{\frac{a-1}{a}}\left(\mathbb{E}|x(s)-x(t_{i+1})|^{a(\alpha-\varepsilon)}\right)^{\frac{1}{a}}\\
\le\ &3\left(2^{\frac{1}{a-1}}\right)\left(\mathbb{E}|x(s)|^{(2+\gamma)\frac{a}{a-1}}+\mathbb{E}|x(t_{i+1})|^{(2+\gamma)\frac{a}{a-1}}\right)^{\frac{a-1}{a}}\left(C|s-t_{i+1}|^{\frac{a(\alpha-\varepsilon)}{2}}\right)^{\frac{1}{a}}\\
\le\ &C|s-t_{i+1}|^{\frac{\alpha-\varepsilon}{2}}.
\end{aligned}$$

So we can get from (**??**) that

$$\begin{aligned}
&\mathbb{E}(A_1)\\
\le\ &\int_{t_i}^{t_{i+1}}\left[(K_3+\frac{1}{2})\mathbb{E}|x(t_{i+1})-y_{i+1}|^2+\frac{K_2}{2}\mathbb{E}\left((1+|x(s)|^\gamma+|x(t_{i+1})|^\gamma)|x(s)-x(t_{i+1})|^2\right)\right]ds\\
\le\ &\int_{t_i}^{t_{i+1}}\left[(K_3+\frac{1}{2})\mathbb{E}|x(t_{i+1})-y_{i+1}|^2+C\Delta^{\frac{\alpha-\varepsilon}{2}}\right]ds\\
\le\ &(K_3+\frac{1}{2})\Delta\mathbb{E}|x(t_{i+1})-y_{i+1}|^2+C\Delta^{\frac{\alpha-\varepsilon}{2}+1}.
\end{aligned} \tag{3.3}$$

Now we are handing the A_2. By the elementary inequality, we can show that

$$A_2 = \frac{1}{2}|x(t_{i+1})-y_{i+1}|^2+\frac{1}{2}\left|(x(t_i)-y_i)+\int_{t_i}^{t_{i+1}}(g(x(s))-g(y_i))\,dB(s)\right|^2.$$

Define

$$A_{21} = \left|(x(t_i)-y_i)+\int_{t_i}^{t_{i+1}}(g(x(s))-g(y_i))\,dB(s)\right|^2.$$

Taking expectation on both sides, by the Itô isometry we have

$$\mathbb{E}(A_{21}) = \mathbb{E}|(x(t_i)-y_i)|^2+\int_{t_i}^{t_{i+1}}|g(x(s))-g(y_i)|^2\,ds.$$

Rewriting the integrand of the second term on he right hand side, by the elementary inequality and (??), we then have

$$\begin{aligned}|g(x(s))-g(y_i)|^2 &\leq 2|g(x(s))-g(x(t_i))|^2+2|g(x(t_i))-g(y_i)|^2\\ &\leq 2K_2\left(|x(s)-x(t_i)|^2+|x(t_i)-y_i|^2\right).\end{aligned}$$

We choose $\overline{a}=(2\alpha-\varepsilon)/(2\alpha-2\varepsilon)$, then we have $(2-\alpha+\varepsilon)\frac{\overline{a}}{\overline{a}-1}\geq 2$ and $1<\overline{a}(\alpha-\varepsilon)<\alpha$. By Hölder's inequality, the elementary inequality, (??) and (??),

$$\begin{aligned}\mathbb{E}|x(s)-x(t_i)|^2 &= \mathbb{E}\left(|x(s)-x(t_i)|^{2-\alpha+\varepsilon}|x(s)-x(t_i)|^{\alpha-\varepsilon}\right)\\ &\leq \left(\mathbb{E}|x(s)-x(t_i)|^{(2-\alpha+\varepsilon)\frac{\overline{a}}{\overline{a}-1}}\right)^{\frac{\overline{a}-1}{\overline{a}}}\left(\mathbb{E}|x(s)-x(t_i)|^{\overline{a}(\alpha-\varepsilon)}\right)^{\frac{1}{\overline{a}}}\\ &\leq \left(\mathbb{E}|x(s)|^{(2-\alpha+\varepsilon)\frac{\overline{a}}{\overline{a}-1}}+\mathbb{E}|x(t_i)|^{(2-\alpha+\varepsilon)\frac{\overline{a}}{\overline{a}-1}}\right)^{\frac{\overline{a}-1}{\overline{a}}}\left(\mathbb{E}(C|s-t_i|^{\frac{\overline{a}(\alpha-\varepsilon)}{2}})\right)^{\frac{1}{\overline{a}}}\\ &\leq C\varDelta^{\frac{\alpha-\varepsilon}{2}}.\end{aligned}\tag{3.4}$$

We can see that

$$\mathbb{E}(A_2)\leq \frac{1}{2}\mathbb{E}|x(t_{i+1})-y_{i+1}|^2+(\frac{1}{2}+K_2\varDelta)\mathbb{E}|x(t_i)-y_i|^2+C\varDelta^{\frac{\alpha-\varepsilon}{2}+1}.\tag{3.5}$$

Combining (??) and (??), we get

$$\begin{aligned}&\mathbb{E}|x(t_{i+1})-y_{i+1}|^2\leq \mathbb{E}(A_1)+\mathbb{E}(A_2)\\ &\qquad\leq (K_3\varDelta+\frac{1}{2}\varDelta+\frac{1}{2})\mathbb{E}|x(t_{i+1})-y_{i+1}|^2+(K_2\varDelta+\frac{1}{2})\mathbb{E}|x(t_i)-y_i|^2+C\varDelta^{\frac{\alpha-\varepsilon}{2}+1}.\end{aligned}$$

Putting all the terms at $t=t_{i+1}$ on the left hand side, we have

$$\mathbb{E}|x(t_{i+1})-y_{i+1}|^2\leq \frac{1+2K_2\varDelta}{1-2K_3\varDelta-\varDelta}\mathbb{E}|x(t_i)-y_i|^2+C\varDelta^{\frac{\alpha-\varepsilon}{2}+1}.$$

Now summing both sides from 1 to i yields

$$\sum_{r=1}^{i}\mathbb{E}|x(t_r)-y_r|^2\leq \frac{1+2K_2\varDelta}{1-2K_3\varDelta-\varDelta}\sum_{r=0}^{i-1}\mathbb{E}|x(t_r)-y_r|^2+iC\varDelta^{\frac{\alpha-\varepsilon}{2}+1}.$$

By $i\varDelta=t_i\leq e^{Ct_i}$, we have

$$\mathbb{E}|x(t_i)-y_i|^2\leq \frac{\varDelta(1+2K_2+2k_3)}{1-2K_3\varDelta-\varDelta}\sum_{r=0}^{i-1}\mathbb{E}|x(t_r)-y_r|^2+C\varDelta^{\frac{\alpha-\varepsilon}{2}}e^{Ct_i}.$$

By discrete Gronwall's inequality we get

$$\mathbb{E}|x(t_i)-y_i|^2\leq C\varDelta^{\frac{\alpha-\varepsilon}{2}}e^{Ct_i}.\tag{3.6}$$

Moreover, when $t\in[i\varDelta,(i+1)\varDelta)$ for any $i=1,2,\cdots$, by (??), (??) and (??) we have

$$\mathbb{E}|x(t)-y(t)|^2\leq 2\mathbb{E}|x(t)-x(t_i)|^2+2\mathbb{E}|x(t_i)-y_i|^2\leq C\varDelta^{\frac{\alpha-\varepsilon}{2}}.$$

The proof is completed. □

4 Example

Consider a scalar SDE

$$\mathrm{d}x(t) = (x(t) - x^3(t))\mathrm{d}t + 2x(t)\mathrm{d}B(t) + \mathrm{d}L(t), \tag{4.1}$$

with the initial value $x(0) = x_0$. It is clear that for any $q \geq 2$ (**A1**) holds with

$$x(x - x^3) + \frac{q-1}{2}|2x|^2 \leq (2q-3)(1 + |x|^2).$$

Moreover, we have

$$|(x - x^3) - (y - y^3)|^2 \leq 8|x - y|^2(1 + |x|^4 + |y|^4) \text{ and } |2x - 2y|^2 \leq 4|x - y|^2,$$

where Young's inequality $|x|^2|y|^2 \leq |x|^4 + |y|^4$ is used. Thus, (**A2**) is satisfied. In addition,

$$(x - y)((x - x^3) - (y - y^3)) \leq |x - y|^2.$$

That is, (**A3**) is satisfied. Therefore, we conclude by Theorem **??** that the semi-implicit EM is convergent to (**??**) with the rate of $(\alpha - \varepsilon)/4$ for any arbitrarily small $\varepsilon > 0$.

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